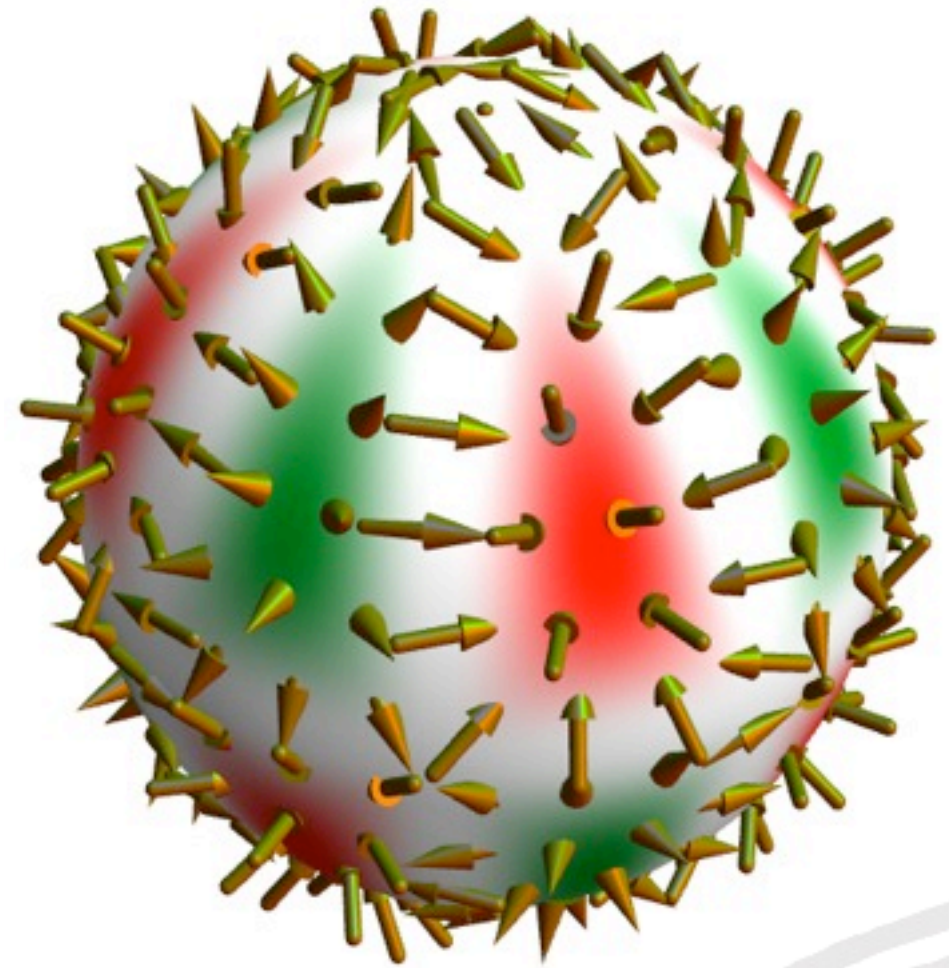




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Lars Nordström  
Uppsala U, Sweden



# DFT+ $U$ method with LAPW

Electronic Structure with the *Elk* Code - CECAM July 2011



# Outline

- DFT+ $U$ 
  - Double counting
- Tensor moments
- General polarisation of the open shell
- Katt's rules
- Applications
  - URu<sub>2</sub>Si<sub>2</sub> & NpO<sub>2</sub> – uncover the hidden order ...
  - Fe pnictides – why low moments?



# DFT+ $U$

$$U: \quad E_{\text{HF}} = \frac{1}{2} \sum_{abcd} [\rho_{ac}\rho_{bd} - \rho_{ad}\rho_{bc}] \langle ab|g|cd \rangle$$

where  $a, b, c, d$  enumerate the orbitals of the correlated atomic shell ( $d$  or  $f$ )

Problem:

- Part of the interaction is treated both in DFT and  $U$ !
- Have to subtract the *Double Counting*.



# DFT+U

## Electron interaction in terms of Slater parameters

$$\langle ab|g|cd\rangle = \delta(s_a, s_c)\delta(s_b, s_d)(2\ell + 1)^2 \sum_{kq} \begin{pmatrix} \ell & k & \ell \\ 0 & 0 & 0 \end{pmatrix}^2 \\ \times (-)^{m_a+m_b+q} \begin{pmatrix} \ell & k & \ell \\ -m_a & -q & m_c \end{pmatrix} F^{(k)} \begin{pmatrix} \ell & k & \ell \\ -m_b & q & m_d \end{pmatrix}$$

## Bare interactions

$$F^{(k)} = \int dr_1 r_1^2 R_\ell^2(r_1) \frac{r_{<}^k}{r_{>}^{k+1}} R_\ell^2(r_2) r_2^2 dr_2$$

## Screening

$$\frac{r_{<}^k}{r_{>}^{k+1}} \rightarrow -(2k + 1)\lambda j_k(i\lambda r_{<})h_k^{(1)}(i\lambda r_{>})$$





# DFT+ $U$ history

- Main interest - TM oxides ... (Anisimov et al)
- 90 - localisation: only diagonal elements of  $\rho$
- 95 - distortions: block diagonal elements of  $\rho$
- 00 - magnetic order: full  $\rho$  (Solvyev et al)
  - for  $l=2$ ,  $\rho$  has dimension  $10 \times 10$ 
    - no problem for computer ...
    - but for understanding!
- 03 - spherical averaged interaction (Dudarev et al)

# Double counting

- Around mean field (AMF) (Anisimov et al)
  - H and X in DFT correspond to HF for an unpolarised density matrix (DM)
- Fully localised limit (FLL) (Dudarev et al)
  - Appropriate when DM has eigenvalues zero or one only
- Interpolation between the two (Petukhov et al)
  - Interpolation constant calculated from DM

$$E_{\text{DC}} = \alpha E_{\text{DC},\text{FLL}} + (1 - \alpha) E_{\text{DC},\text{AMF}} \quad \alpha = \frac{c_{\text{tot}}}{c_{\text{max}}}$$



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# Background on actinides

- Abundant itinerant magnetic materials
- Unquenched orbital moments & huge anisotropies
- Interplay: magnetism & superconductivity
- DFT overestimates spin moments ...
- Can be remedied in DFT+ approaches

# Polarisation of open shell

Local density matrix for the correlated shell

$$\rho = (2[\ell])^{-1} (n \mathbf{1} \otimes \mathbf{1} + m_z \mathbf{1} \otimes \sigma_z + \dots)$$

$[\ell] \times [\ell]$

$2 \times 2$

Unpolarised

Spin polarisation

$$[\ell] := 2\ell + 1$$

Higher polarisation terms?

Different kinds of orbital polarisation ...

Combined spin & orbital polarisation ...



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# Tensor moments ...

Transformation

$$w^\alpha \equiv w^{kpr} = \text{Tr } \Gamma^{kpr} \rho$$

Racah (42)

NOVEMBER 1 AND 15, 1942

PHYSICAL REVIEW

VOLUME 62

## Theory of Complex Spectra. II

GIULIO RACAH

*The Hebrew University, Jerusalem, Palestine*

Following Dirac's vector model we can also substitute the operator (37) I for the double sign which precedes  $g_k$  and assume as coefficient of  $G^k$  in the exchange interaction between two electrons the expression

$$-[\tfrac{1}{2} + 2(\mathbf{s}_1 \cdot \mathbf{s}_2)](l_1 \| C^{(k)} \| l_2)^2 \sum_r (-1)^r (2r+1) W(l_1 l_1 l_2 l_2; rk) (\mathbf{u}_1^{(r)} \cdot \mathbf{u}_2^{(r)}). \quad (60)$$

It will also be convenient to consider the quantities  $(\mathbf{s}_1 \cdot \mathbf{s}_2)(\mathbf{u}_1 \cdot \mathbf{u}_2)$  as scalar products of "double tensors";<sup>12</sup> a double tensor of the degree  $(\kappa, k)$  is defined as a quantity which behaves as an irreducible tensor of the degree  $\kappa$  with respect to  $\mathbf{S}$  and as an irreducible tensor of the degree  $k$  with respect to  $\mathbf{L}$ . The algebra of these double tensors is a trivial extension of the tensor algebra developed in §3; it must be noted only that such a double tensor does not satisfy the commutation rule (23) with respect to  $\mathbf{J}$ , because with respect to  $\mathbf{J}$  it is reducible and may be decomposed in a sum of irreducible tensors, the degrees of which lie between  $|k - \kappa|$  and  $k + \kappa$ . From this point of view the scalar product  $(\mathbf{l} \cdot \mathbf{s})$  is the scalar part of the decomposition of the double vector  $\mathbf{l}\mathbf{s}$ .





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$$w^\alpha \equiv w^{kpr} = \text{Tr } \Gamma^{kpr} \rho$$

Racah (42)

even  $k$ : density

odd  $k$ : current

$p=0$ : charge

$p=1$ : spin

$r$  coupled  $k$  &  $p$



# Tensor moments ...

Transformation  $\mathbf{w}^\alpha \equiv \mathbf{w}^{kpr} = \text{Tr } \mathbf{\Gamma}^{kpr} \rho$

Racah (42) but with van der Laan & Thole normalisation (95)

even k: density

odd k: current

p=0: charge

p=1: spin

r coupled k & p

jm<sub>j</sub>-basis:

$$\left\{ \mathbf{\Gamma}_t^{kpr} \right\}_{j_1 m_1, j_2, m_2} = \frac{\sqrt{[j_1 j_2]}}{N_{kpr\ell}} (-)^{j_1 - m_1} \begin{pmatrix} j_1 & r & j_2 \\ -m_1 & t & m_2 \end{pmatrix} \left\{ \begin{matrix} \ell & \ell & k \\ s & s & p \\ j_1 & j_2 & r \end{matrix} \right\}$$

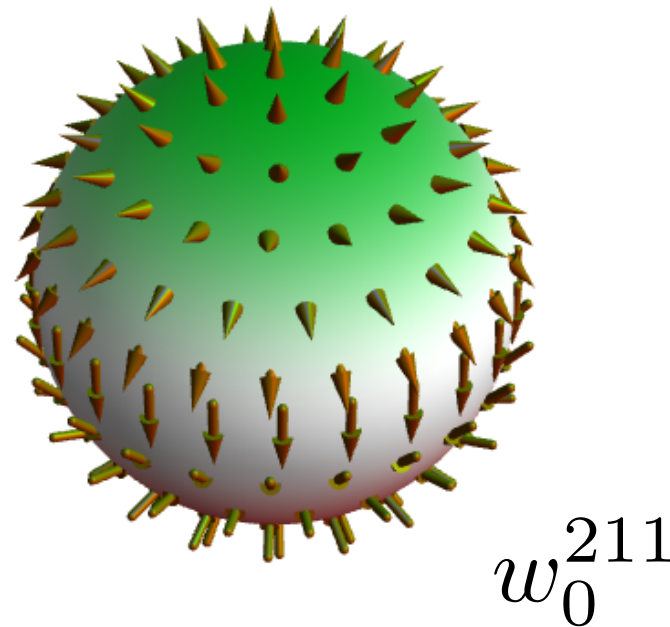


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Racah (42) but with van der Laan & Thole normalisation (95)

even k: density  
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r coupled k & p



$w_0^{211}$

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Racah (42)

but with van der Laan & Thole normalisation (95)

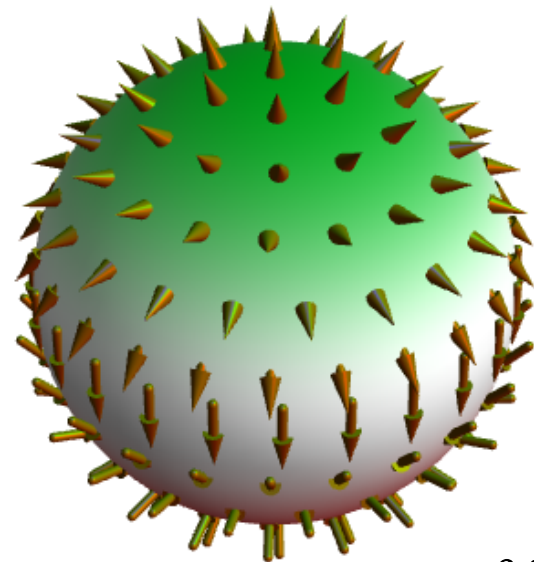
even k: density

odd k: current

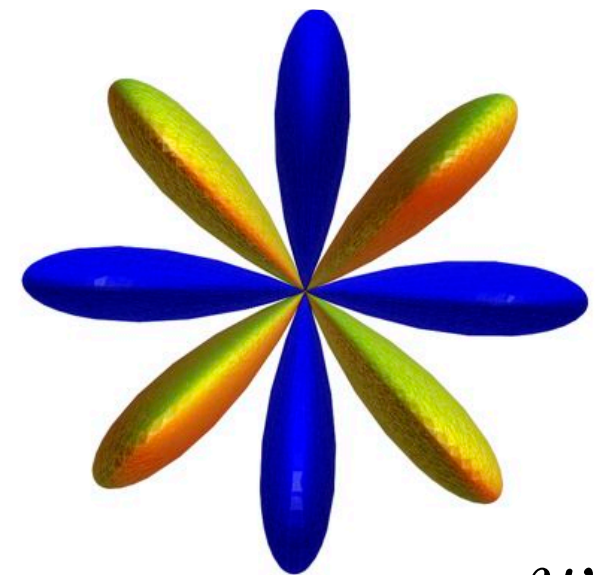
p=0: charge

p=1: spin

r coupled k & p



$w_0^{211}$



$w_{40}^{41}$

jm<sub>j</sub>-basis:

$$\left\{ \mathbf{\Gamma}_t^{kpr} \right\}_{j_1 m_1, j_2, m_2} = \frac{\sqrt{[j_1 j_2]}}{N_{kpr\ell}} (-)^{j_1 - m_1} \begin{pmatrix} j_1 & r & j_2 \\ -m_1 & t & m_2 \end{pmatrix} \left\{ \begin{matrix} \ell & \ell & k \\ s & s & p \\ j_1 & j_2 & r \end{matrix} \right\}$$

# Open shell polarisation

Physical density matrix  $\text{Tr} \rho \geq \text{Tr} \rho^2$

with "polarisation"

$$c^{kpr} = [kpr] |N_{kpr\ell}|^2 \mathbf{w}^{kpr} \cdot \mathbf{w}^{kpr}$$

$$\sum_{kpr \neq 000} c^{kpr} \leq nn_h$$

$$(c_{\text{tot}} \leq c_{\text{max}})$$





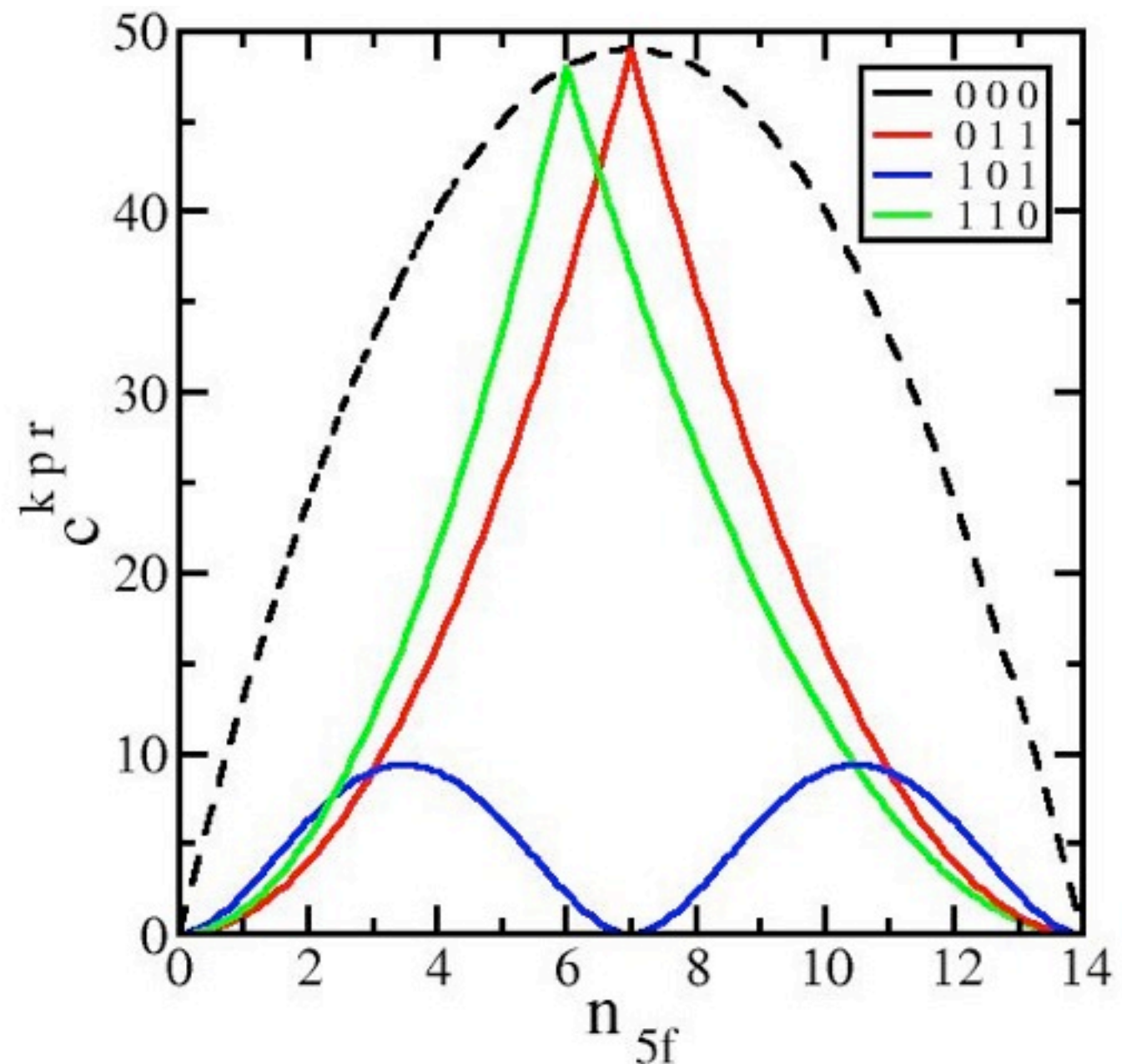
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# Open shell polarisation

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# Reformulation of HF

”GHF” 
$$E_{\text{HF}} = \frac{1}{2} \sum_{abcd} [\rho_{ac}\rho_{bd} - \rho_{ad}\rho_{bc}] \langle ab|g|cd \rangle$$

Transformation dm to spherical tensors (à la Racah)

one-to-one  $\rho \leftrightarrow \mathbf{w}^\alpha$

$$\Rightarrow E_{\text{HF}} = \frac{1}{2} \sum_{\alpha} (J_{\alpha} - K_{\alpha}) \mathbf{w}^{\alpha} \cdot \mathbf{w}^{\alpha}$$



# Reformulation of HF

“GHF”

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# DFT

$$\mathcal{H} = -\Delta + V + \xi \ell \cdot \boldsymbol{\sigma} - b_{\text{XC}} \cdot \boldsymbol{\sigma}$$

$$b_{\text{XC}} \approx \frac{1}{2} I_{\text{Stoner}} m_{\text{spin}}$$

$$\boldsymbol{\sigma} \quad \text{su}(2)$$

$$m_{\text{spin}} \quad m_{\text{orb}}$$

6 degrees of freedom





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# DFT+U

Anisimov, Liechtenstein, Solovyev, Shick ...

$$\mathcal{H} = -\Delta + V + \xi \ell \cdot \boldsymbol{\sigma} - b_{\text{XC}} \cdot \boldsymbol{\sigma} - \sum_{\alpha} B^{\alpha} \cdot \boldsymbol{\Gamma}^{\alpha}$$

$$b_{\text{XC}} \approx \frac{1}{2} I_{\text{Stoner}} m_{\text{spin}}$$

$$\boldsymbol{\sigma} \quad \text{su}(2)$$

$$m_{\text{spin}} \quad m_{\text{orb}}$$

6 degrees of freedom

$$B^{\alpha} = K_{\alpha} w^{\alpha}$$

$$\boldsymbol{\Gamma}^{\alpha} \quad \text{su}(14)$$

$$w^{\alpha}$$

195 dof





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# DFT+ $U$ vs Hybrid-DFT

- Both add exact exchange
- DFT+ $U$  restricts to the correlated shell
- DC vs reduced DFT
- HDFT: "screen" everything with 0.25
  - OK for  $U$  ...
  - But not for higher Slater parameters, that are screened much less (0.7 – 0.9)
- DFT+ $U$  as a CDFT ...?

# Free parameters ...

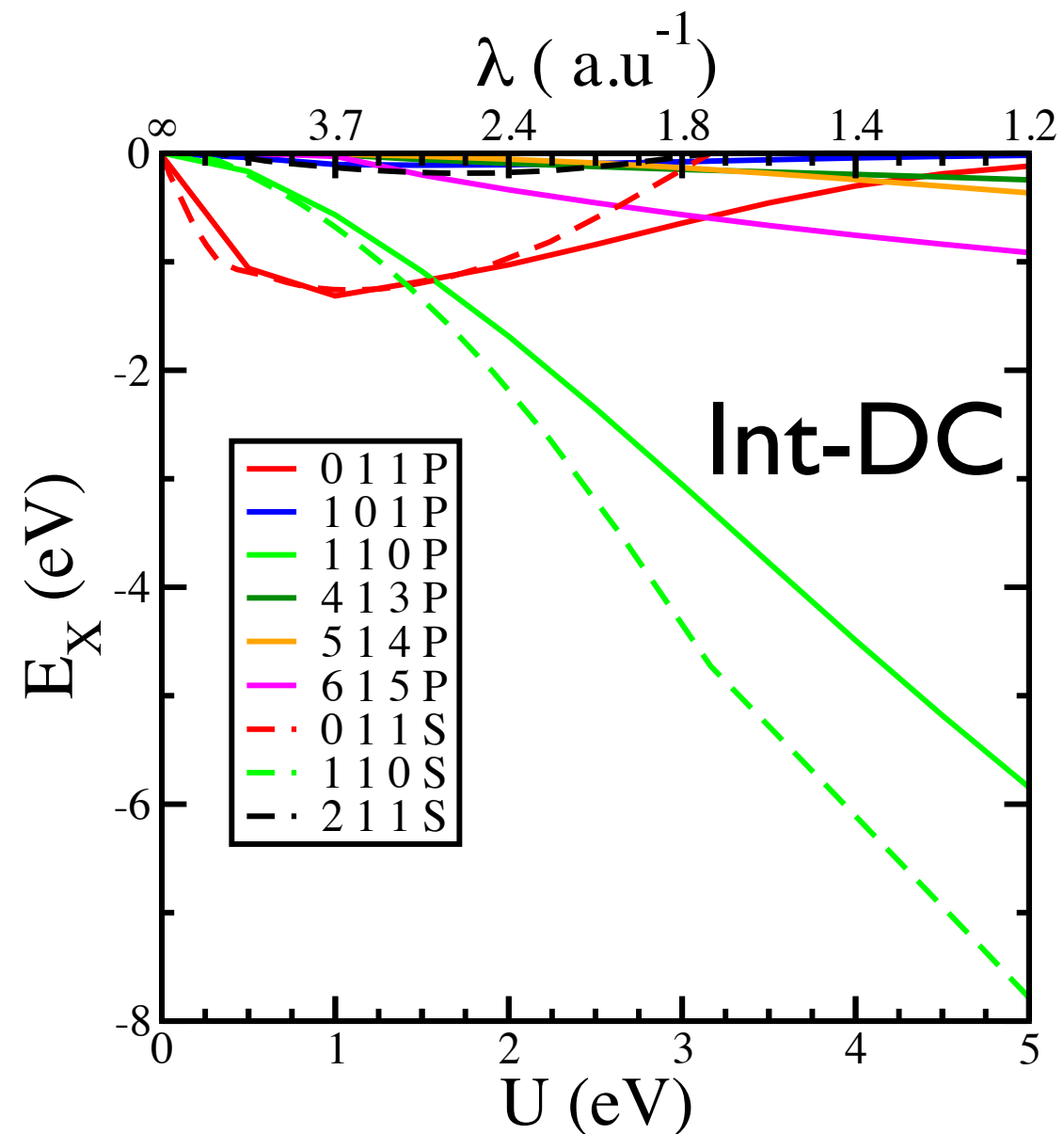
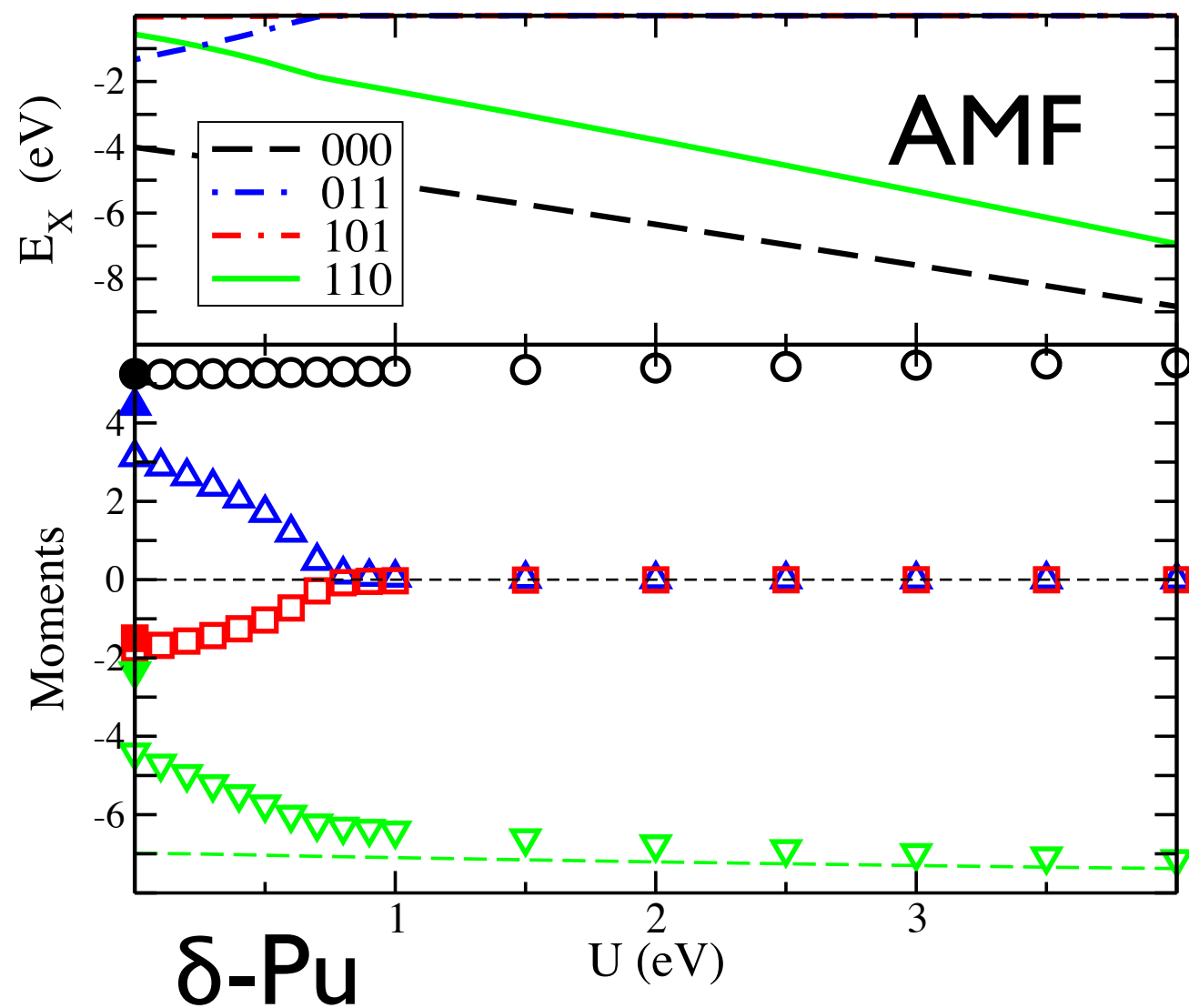
- In the fields  $B^\alpha$  – 4 Slater integrals:  $F^{(k)}$ 
  - Calculate with Yukawa potential  $\frac{1}{r}e^{-\lambda r}$ 
    - One parameter –  $U = F^{(0)}$
- Double counting – AMF or FLL?
  - Interpolation scheme by Petukhov *et al*
- Implemented in ELK (FP-APW+lo)  
un-constrained non-collinear  
[elk.sourceforge.net](http://elk.sourceforge.net) (open source)





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# Non-magnetic Pu



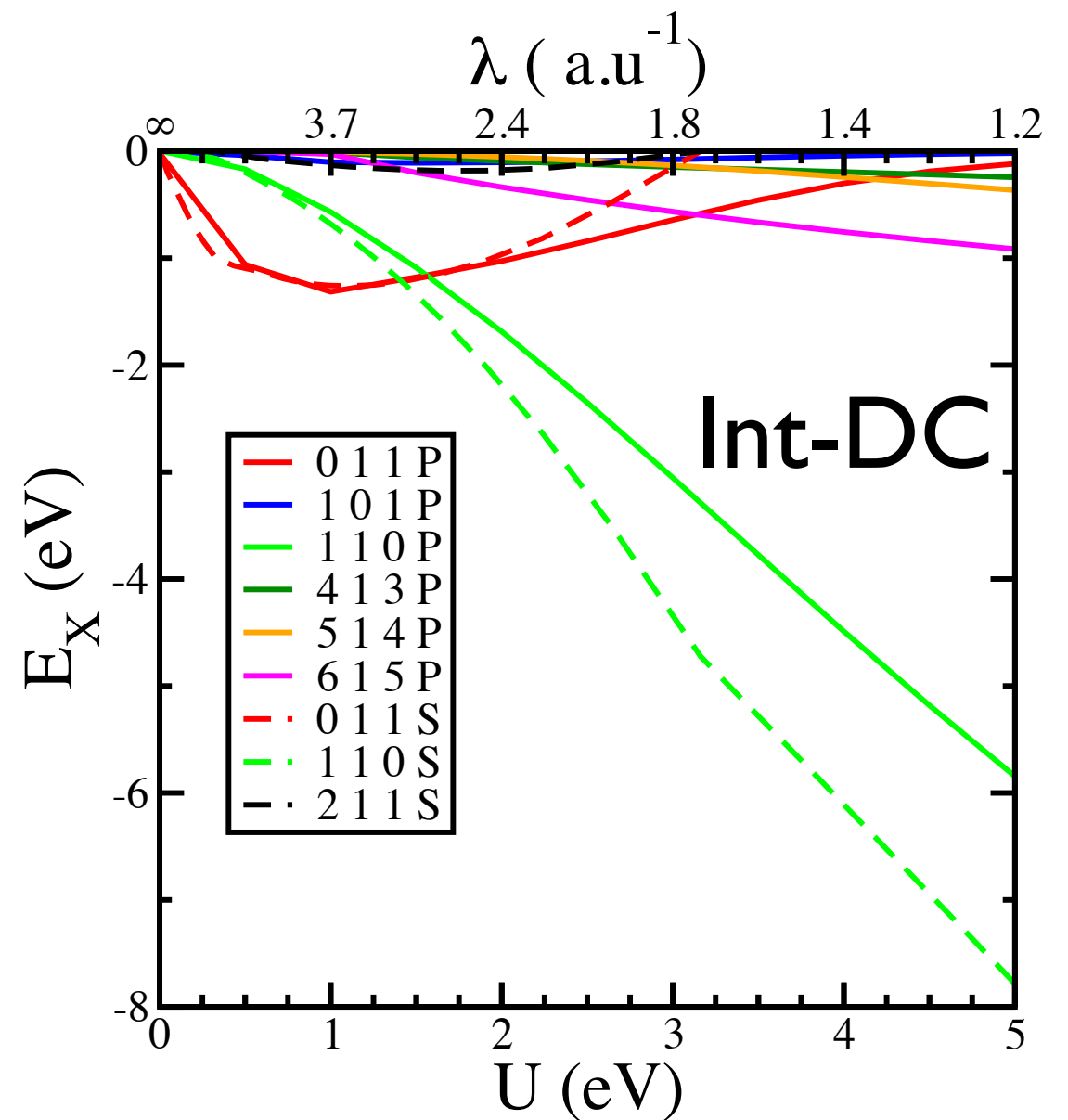
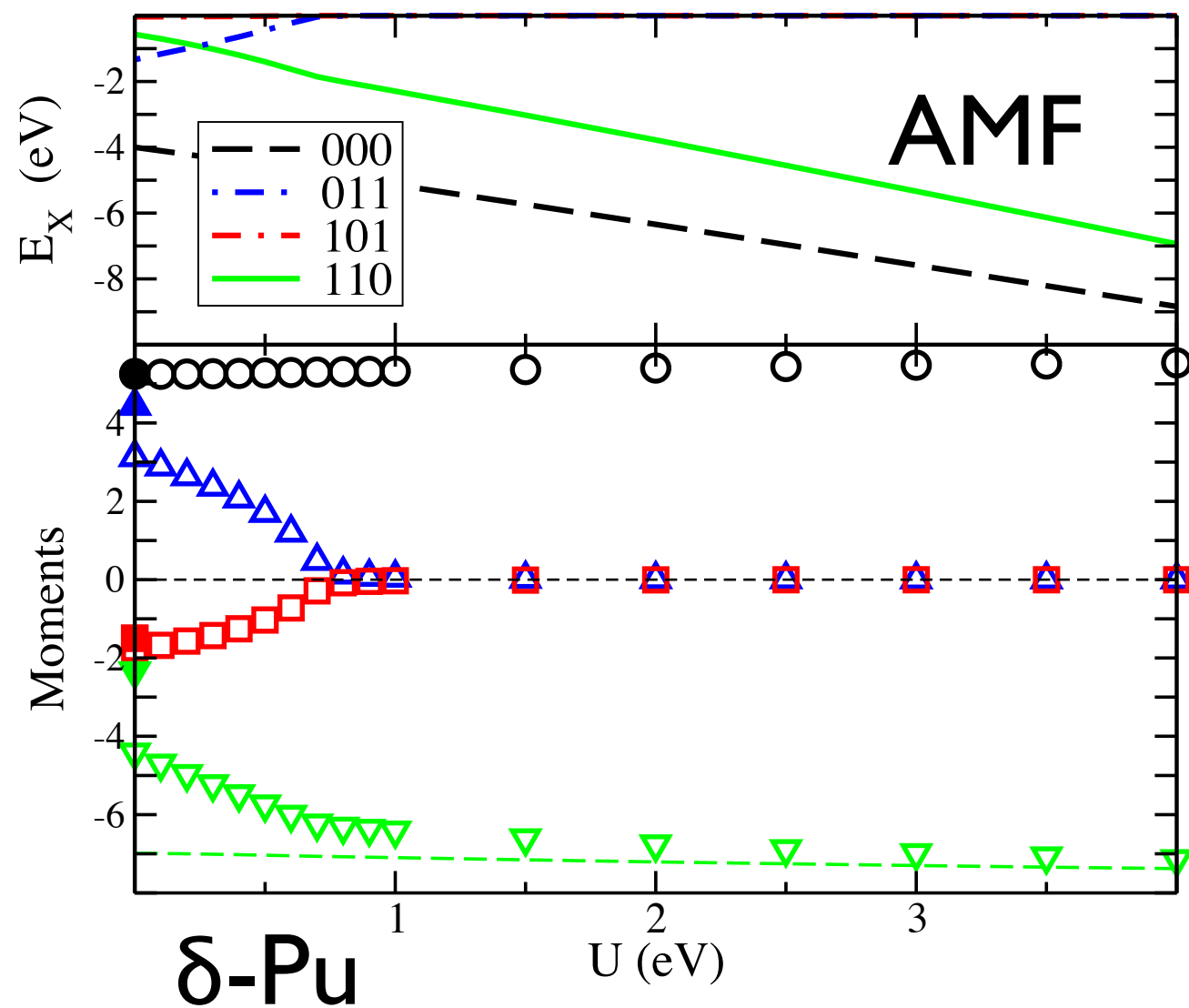
PuS & PuP in NaCl, PM resp FM

Exchange enhanced SOC!



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# Non-magnetic Pu



PuS & PuP in NaCl, PM resp FM

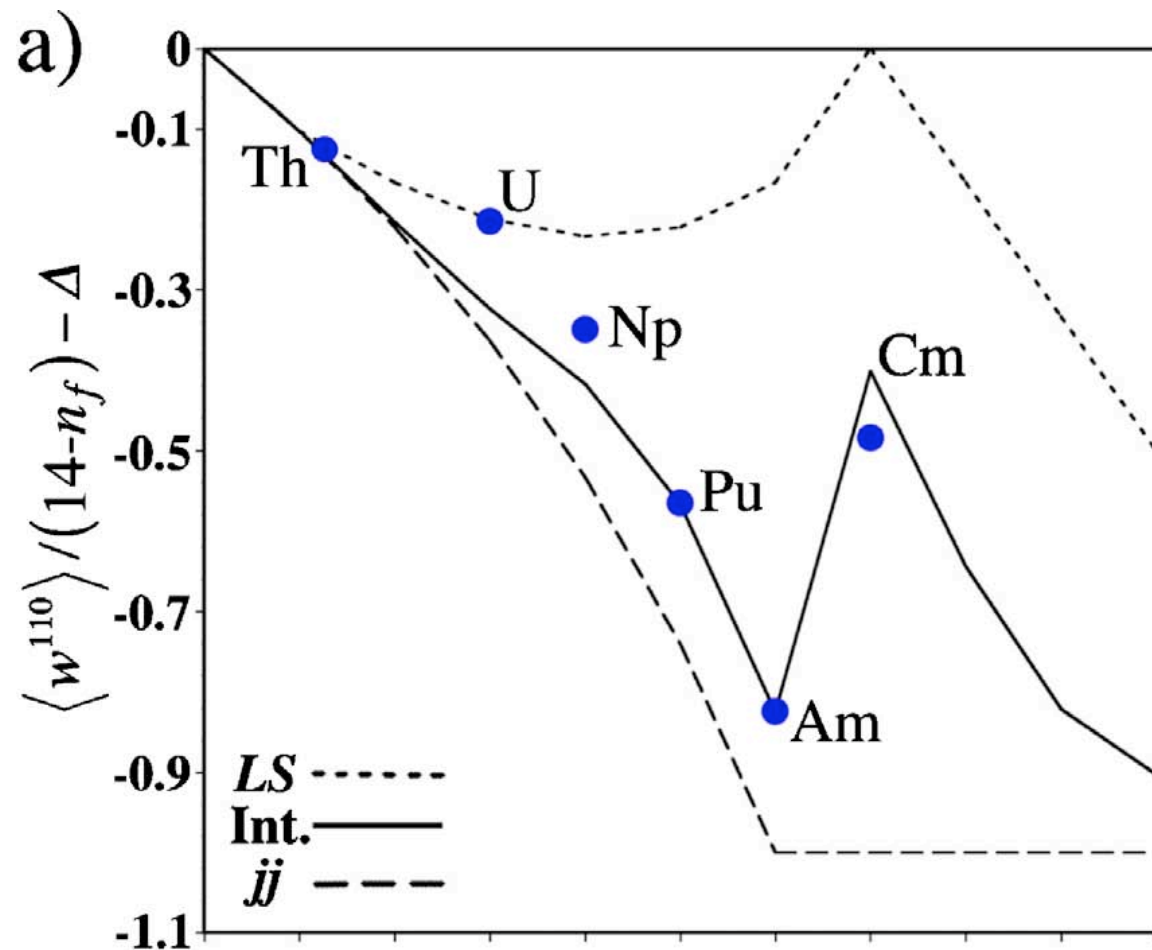
Cricchio *et al*, PRB **78**, 100404(R) (2008) & Bultmark *et al*, PRB **80**, 035121 (2009)

Exchange enhanced SOC!



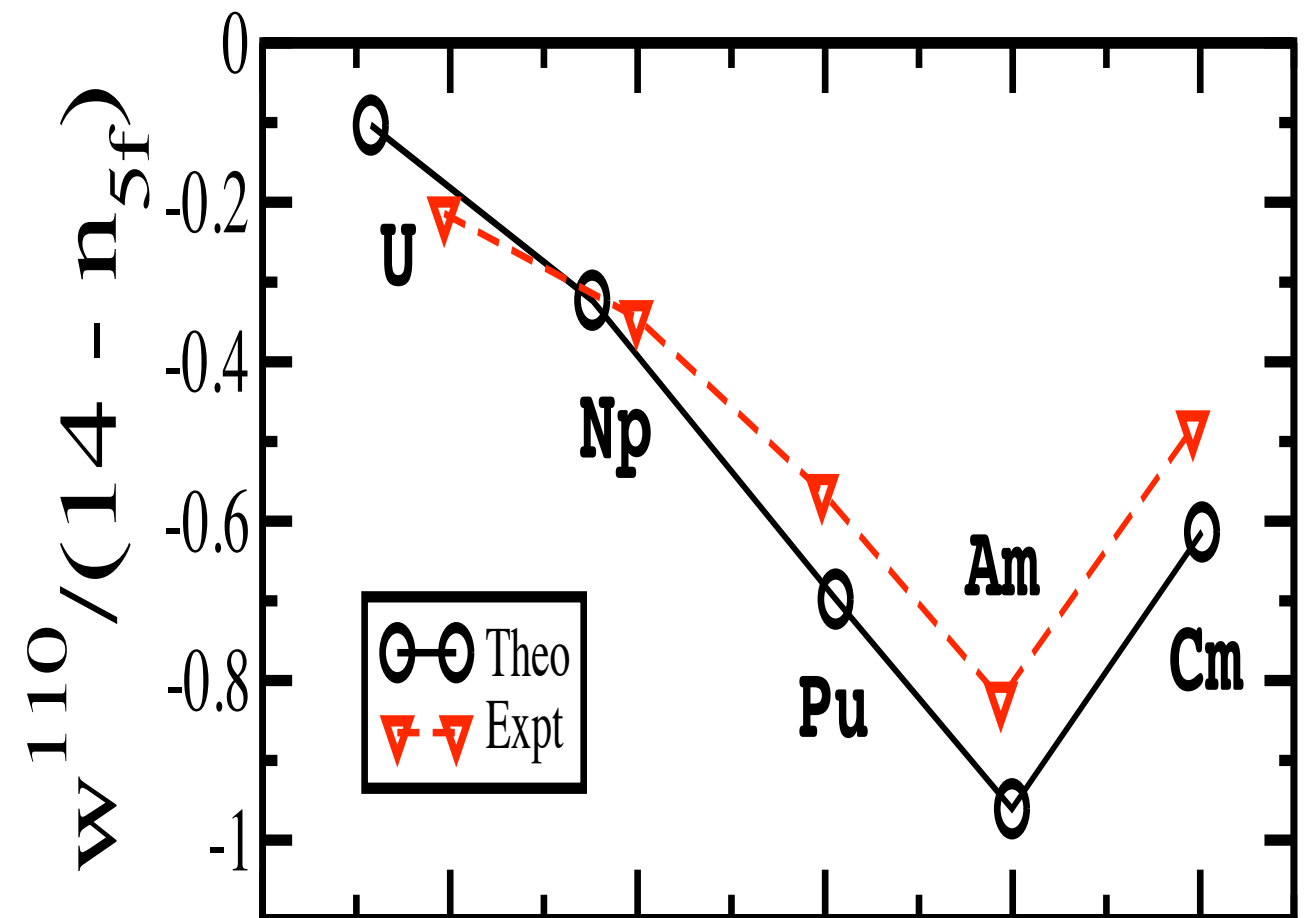
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# 110 moments



EELS SO-sum rule

from Moore et al, PRB **76**, 073105 (2007)



present work

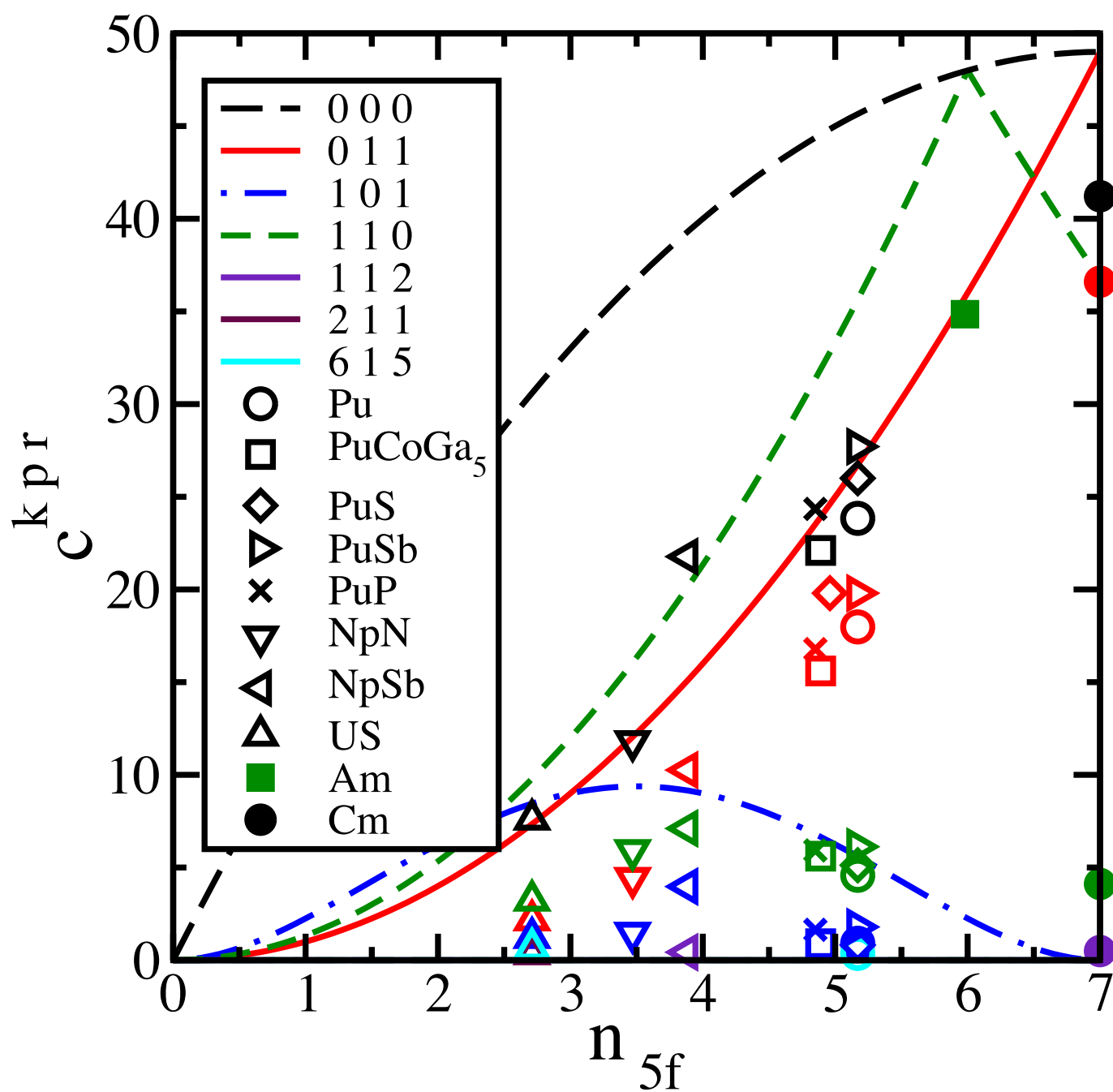




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# Polarisation terms

LDA



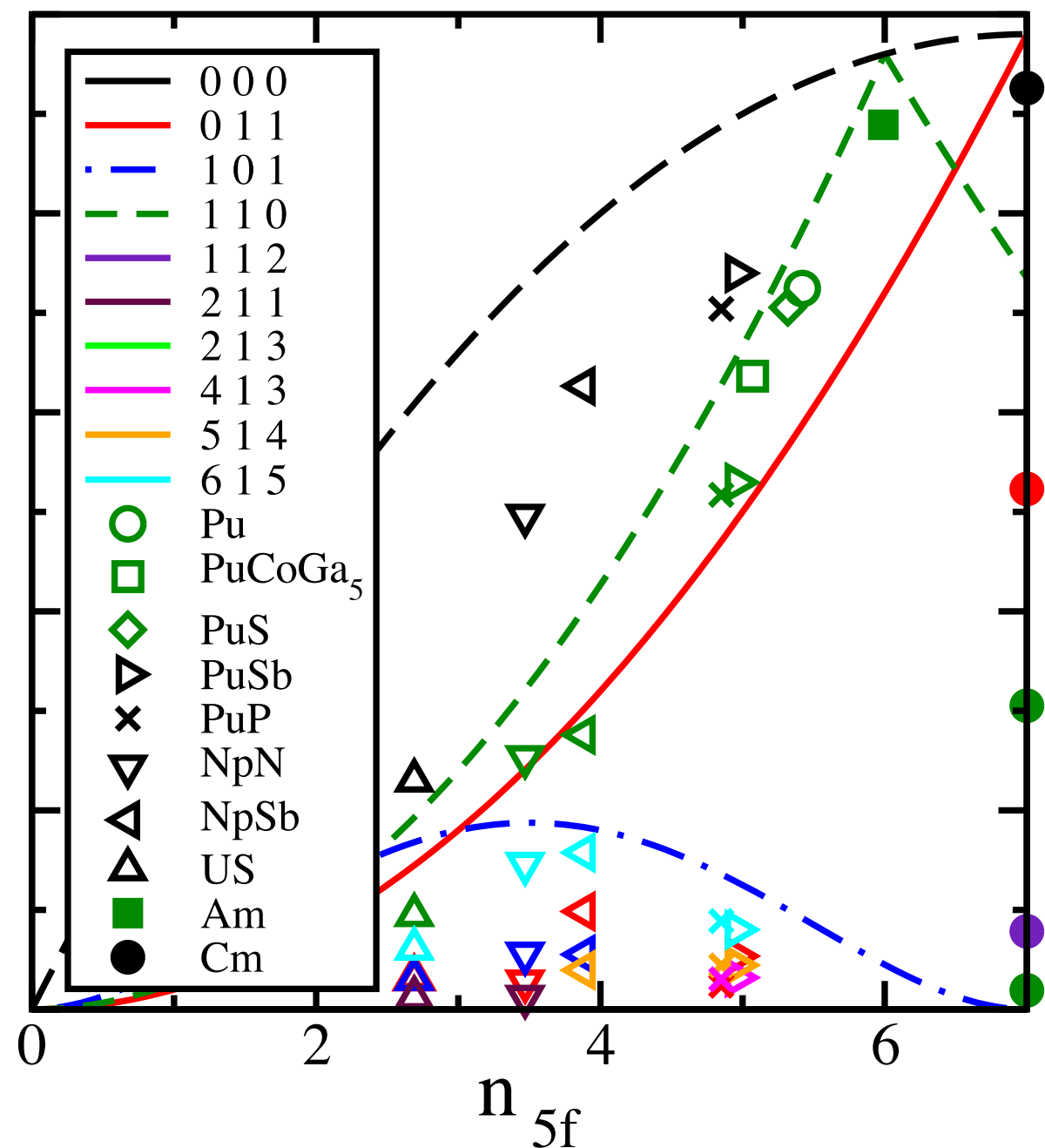
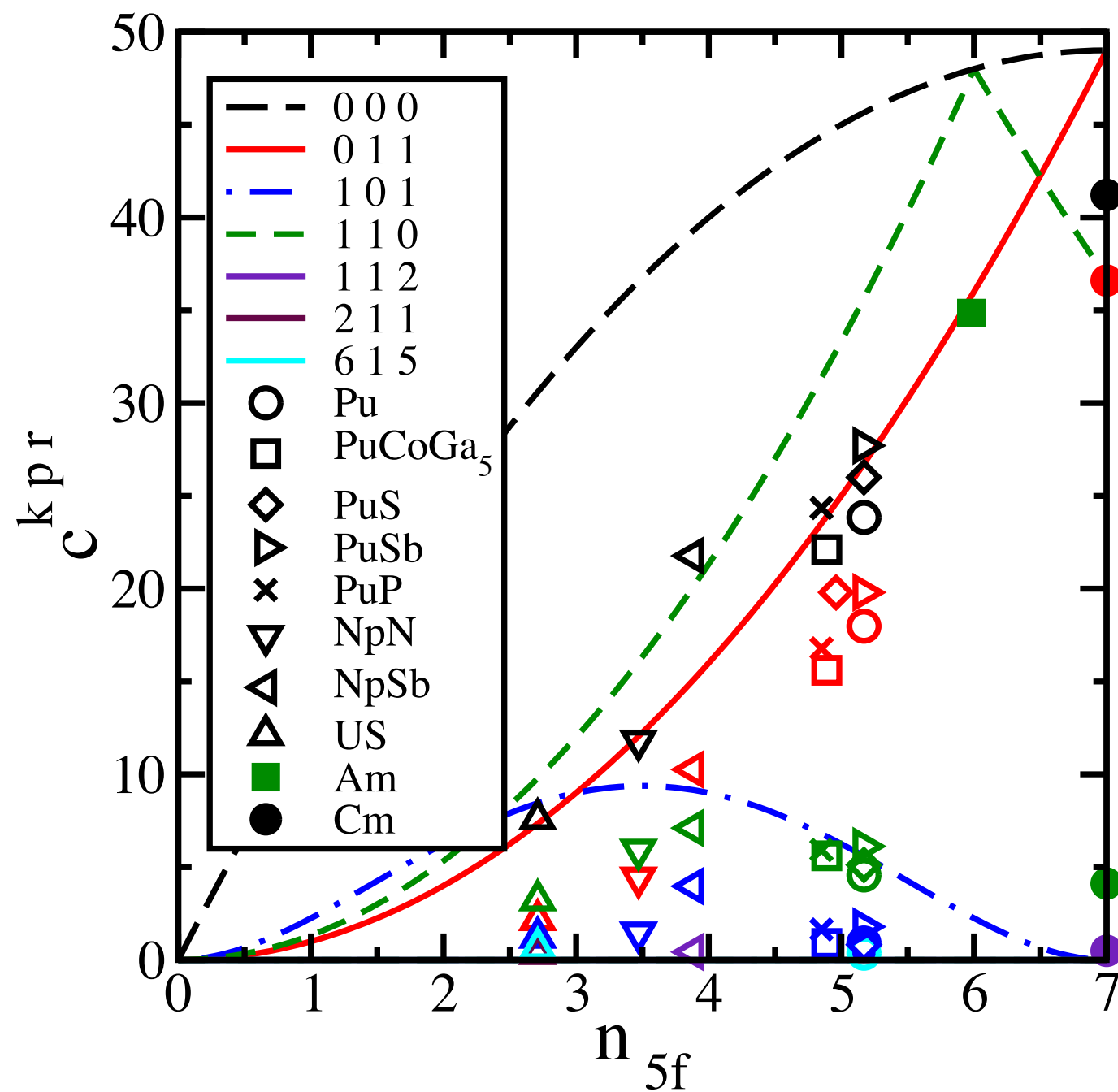


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# Polarisation terms

LDA

LDA+U





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# Time reversal symmetry

First: enhance spin  
orbit coupling  
 $\Rightarrow 110$

$$\left( \begin{array}{cccccccccccccccc} -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} 5/2 \\ \\ \\ \\ \\ \\ \\ 7/2 \end{array}$$



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# Time reversal symmetry

First: enhance spin  
orbit coupling  
 $\Rightarrow 110$

$$\begin{pmatrix} -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Second: break time  
reversal symmetry  
 $\Rightarrow 615$

$$615_0 \sim \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$615_4 \sim \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$



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for extended systems:

# Hund's rules

HI. Saturate spin polarisation – 011

HII. Optimise polarisation 101  
Induced polarisations  
110 & 112

HIII. Let  $w^{110} < 0$

# Katt's rules

KI. Saturate spin-orbit polarisation – 110

KII. Optimise polarisation 615 (617)  
Induced polarisations:  
415 & 505 (none)

KIII. Polarise 011, if possible



for extended systems:

# Hund's rules

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Induced polarisations

Induced polarisations:

110 & 112

415 & 505 (none)

HIII. Let  $w^{110} < 0$

KIII. Polarise 011, if possible





# DFT+U

In terms of polarisation

$$E_X = -\frac{1}{4} \sum_{kpr} A_k c^{kpr}$$

$$A_k = \sum_{n'} B_{kn'}^{(\ell)} F^{(n')} = \sum_n C_{kn}^{(\ell)} E^{(n)}$$

In terms of Racah parameters

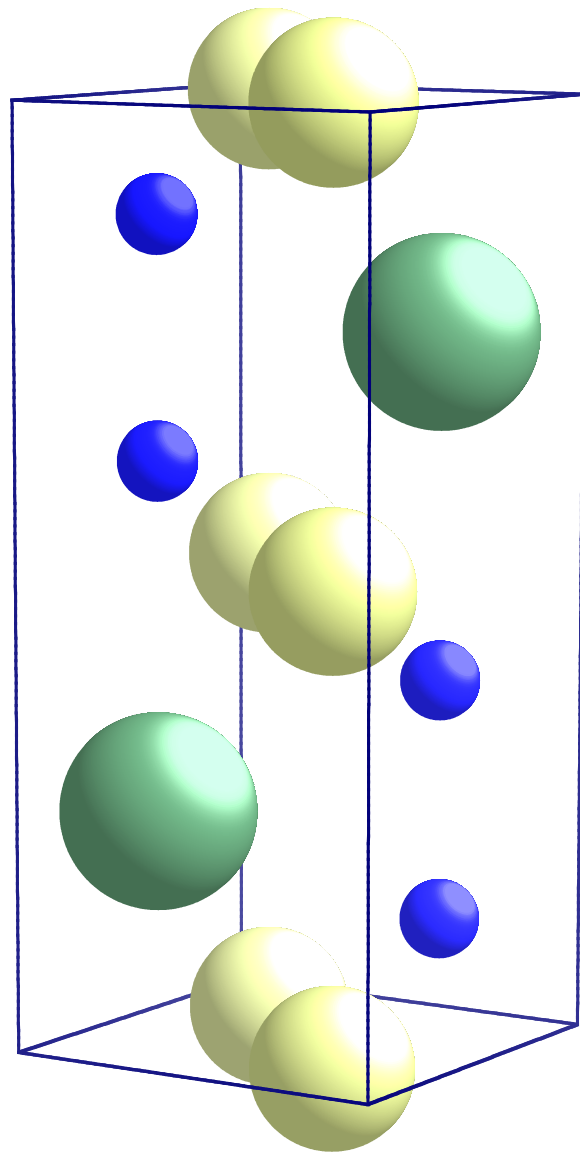
$$D^{(3)} = \begin{pmatrix} 1 & -\frac{2}{45} & -\frac{1}{33} & -\frac{50}{1287} \\ 0 & \frac{14}{405} & \frac{7}{297} & \frac{350}{11583} \\ 0 & \frac{1}{2025} & -\frac{1}{3267} & \frac{175}{1656369} \\ 0 & \frac{1}{135} & \frac{2}{1089} & -\frac{175}{42471} \end{pmatrix} \Rightarrow C^{(3)} = \frac{1}{7} \begin{pmatrix} 1 & 9 & 0 & 0 \\ 1 & 0 & 0 & 33 \\ 1 & 2 & 286 & -11 \\ 1 & 0 & 0 & 0 \\ 1 & 2 & -260 & -4 \\ 1 & 0 & 0 & -9 \\ 1 & 2 & 70 & 7 \end{pmatrix}$$



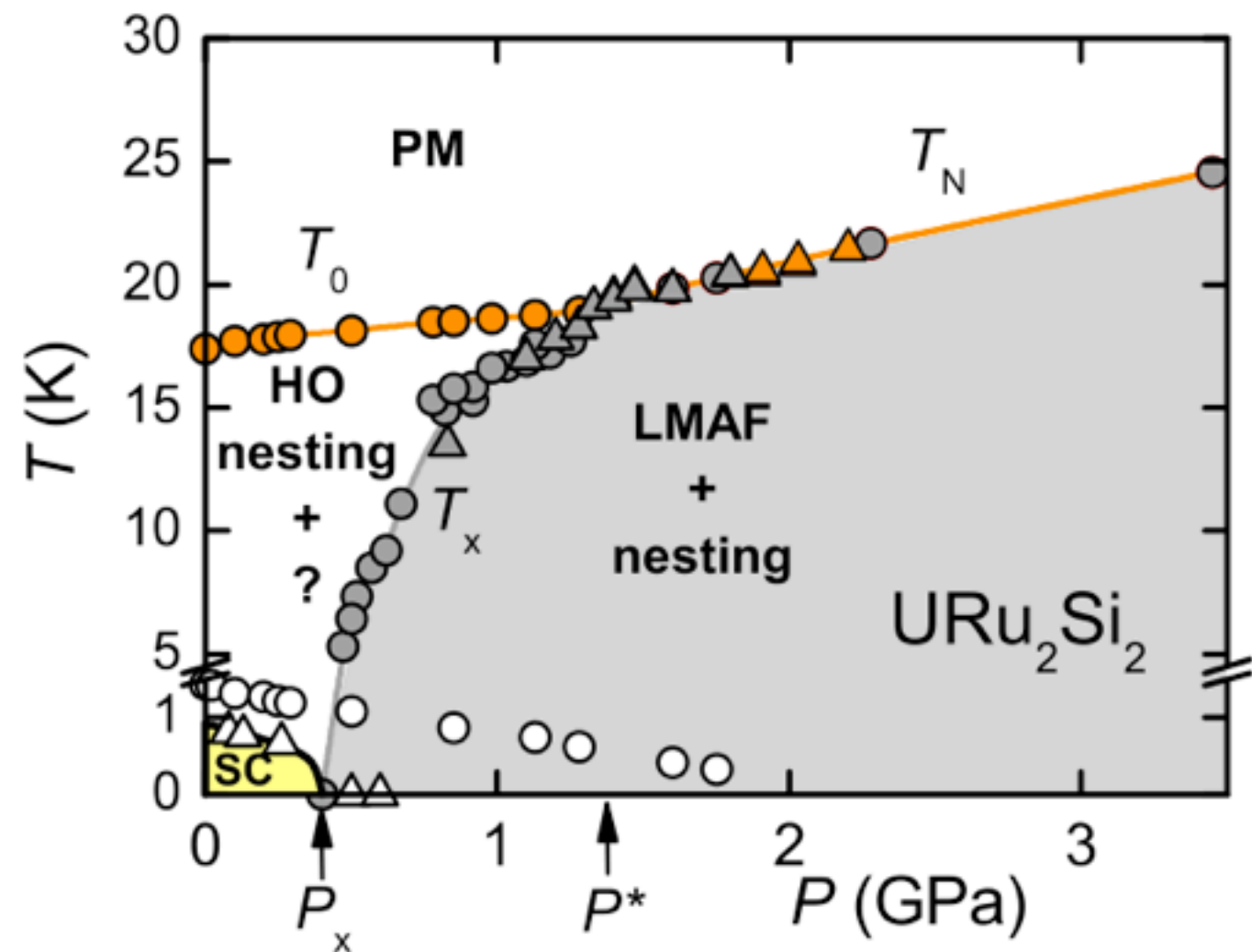
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# Hidden order

**URu<sub>2</sub>Si<sub>2</sub>** heavy fermion material  
Palstra *et al*, 1985



bct

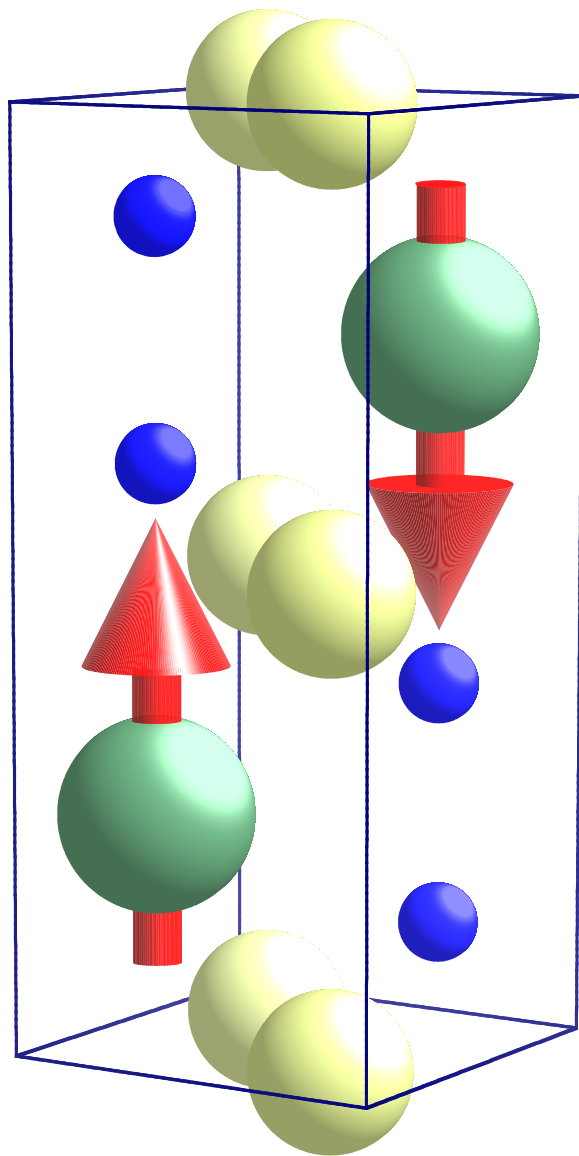




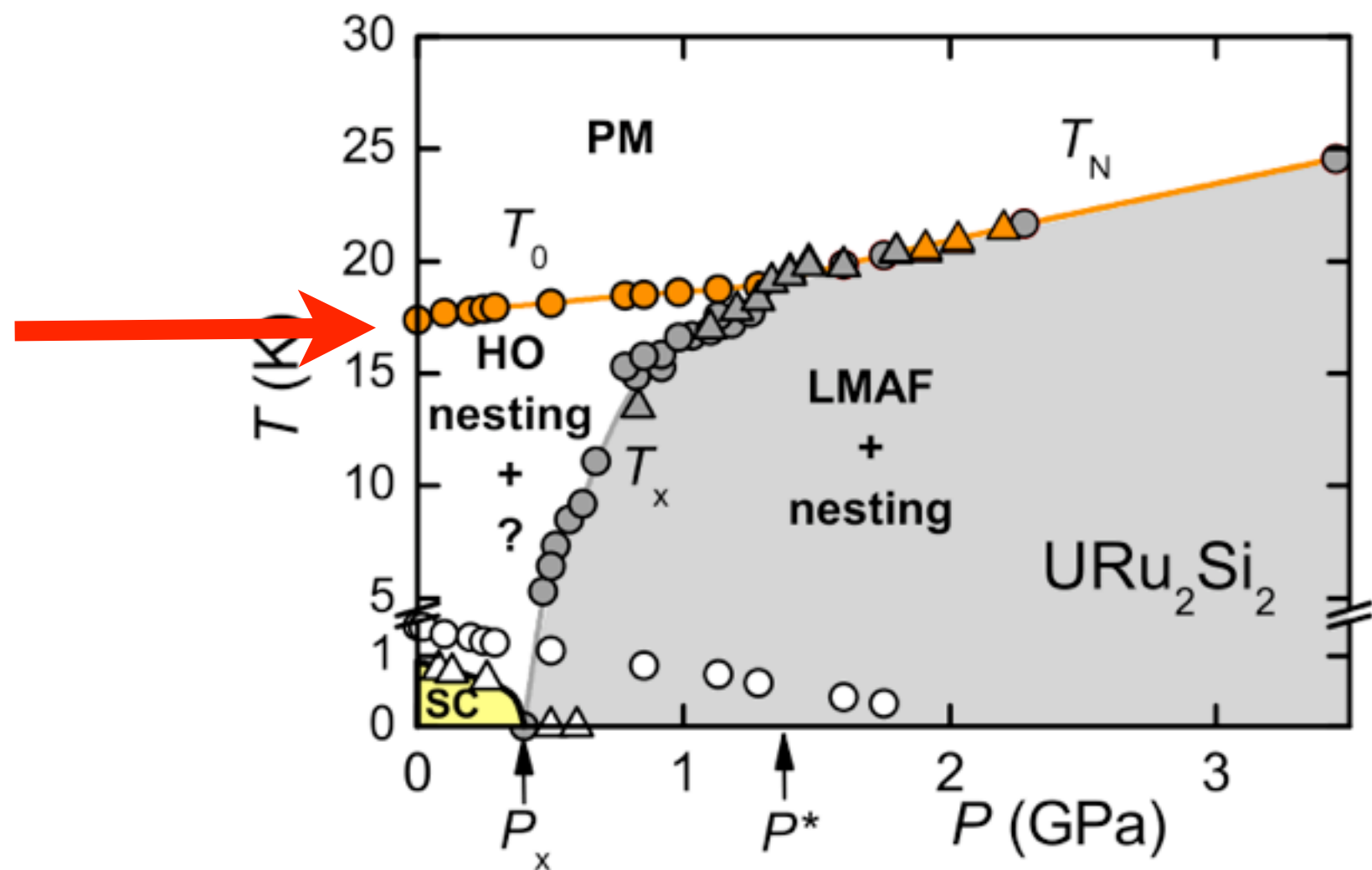
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# Hidden order

$\text{URu}_2\text{Si}_2$  heavy fermion material  
Palstra *et al*, 1985



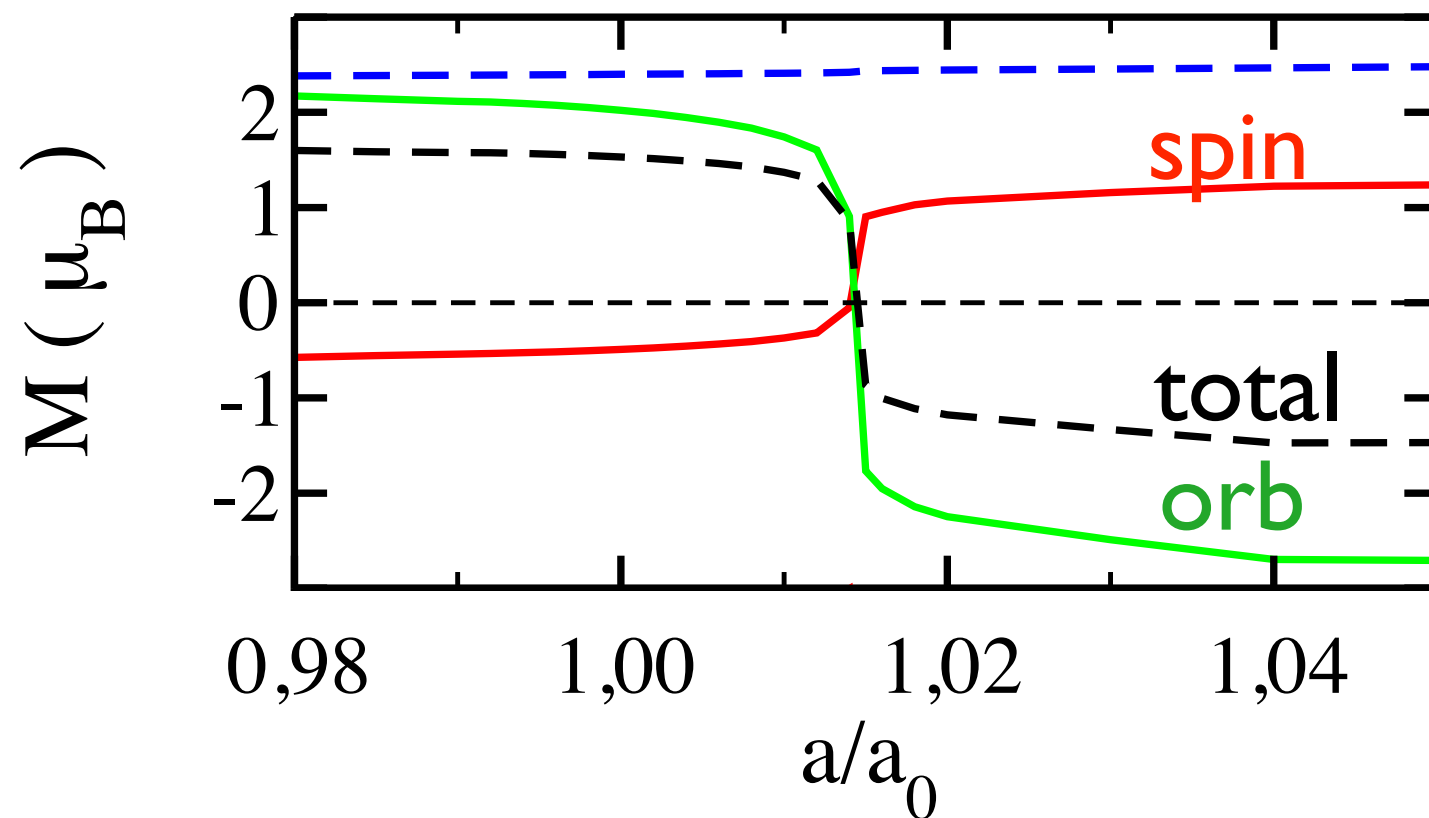
bct





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# DFT+ $U$ calculation



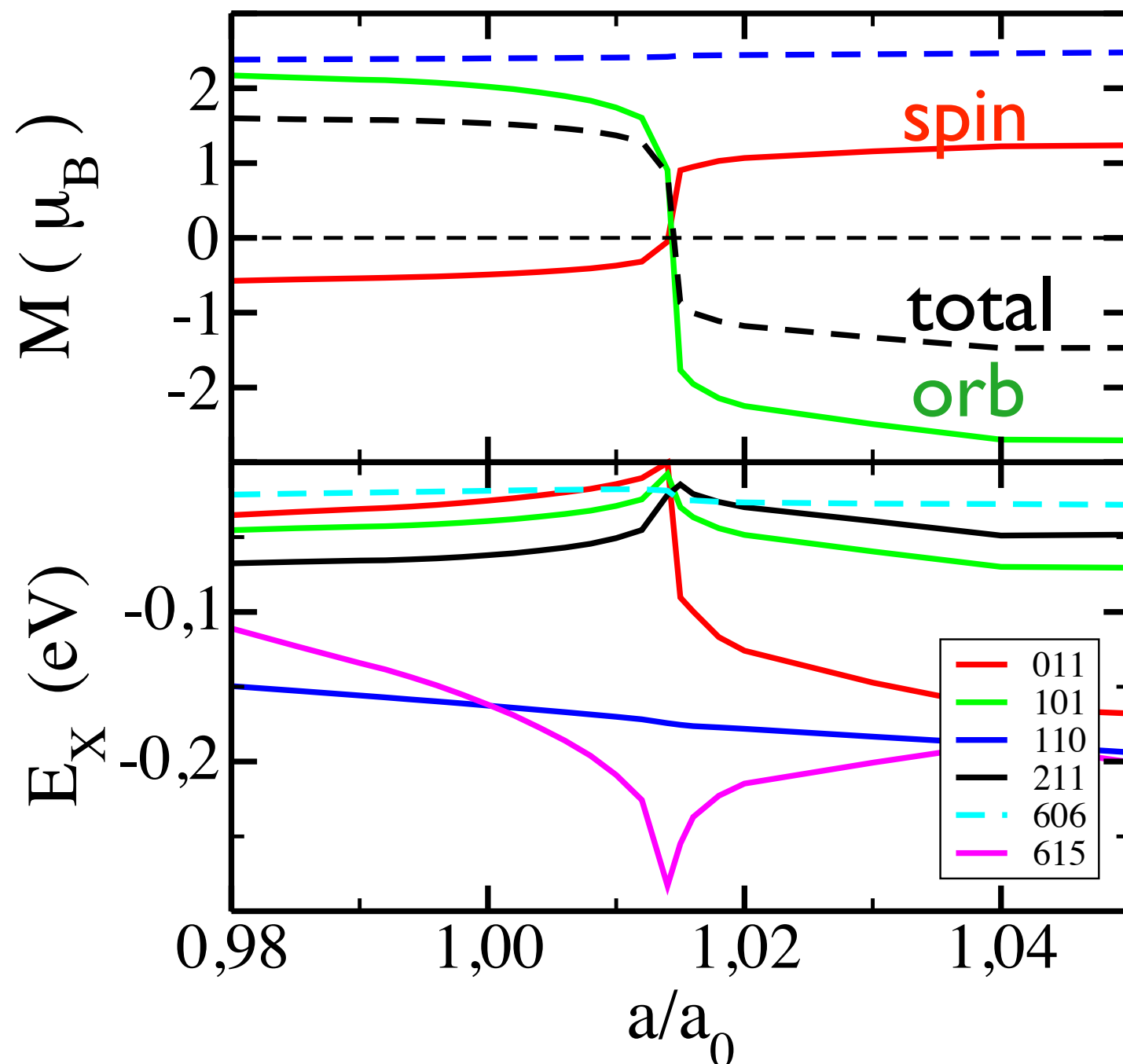
Pressure: mainly  
decrease in  $a$

$$U = 1 \text{ eV}$$



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# DFT+ $U$ calculation



Pressure: mainly  
decrease in  $a$

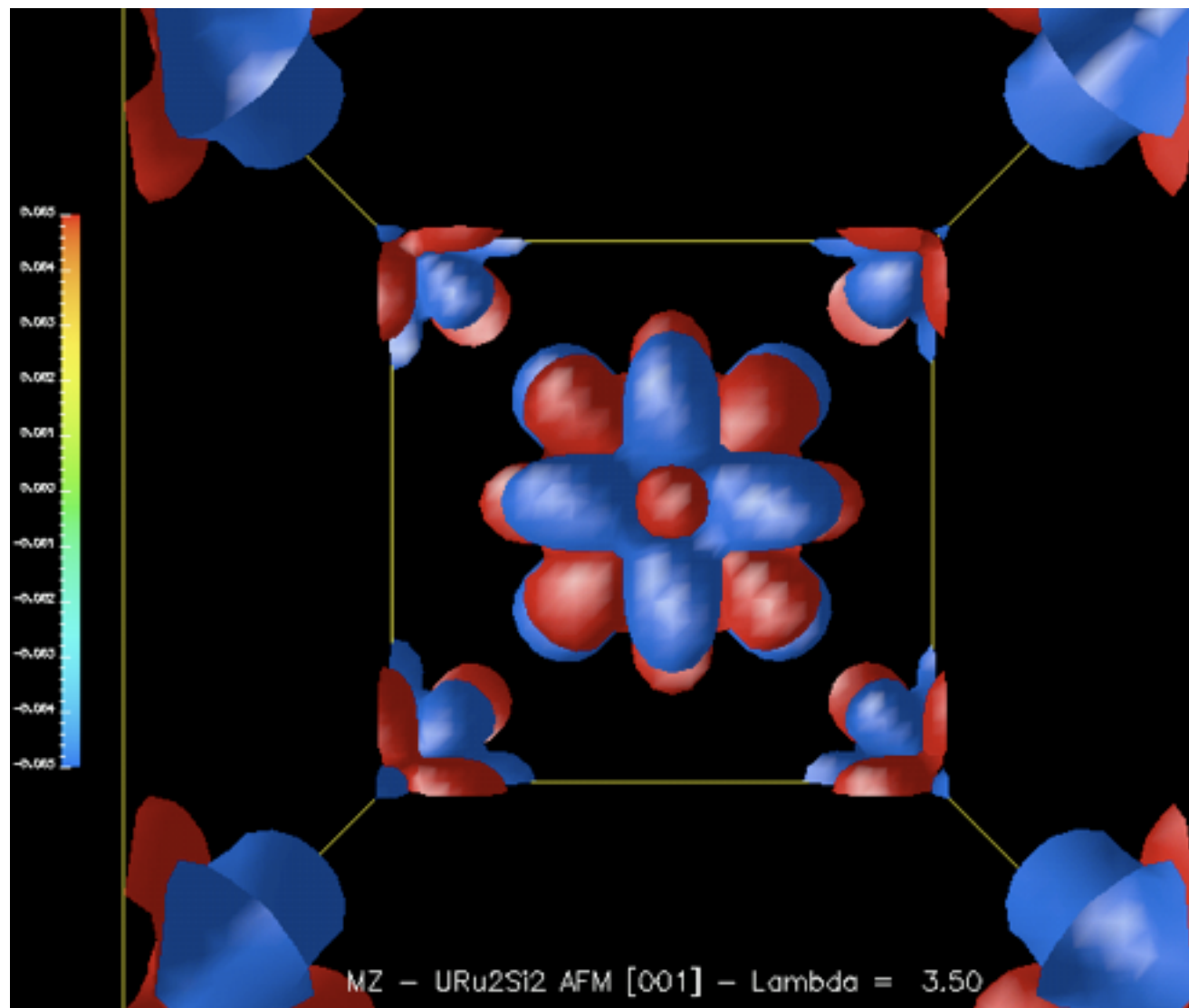
$$U = 1 \text{ eV}$$

Divergence in  
615 exchange  
energy – kills SP!



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# DFT+ $U$ calculation



$m_z$  density  
blue pos & red neg

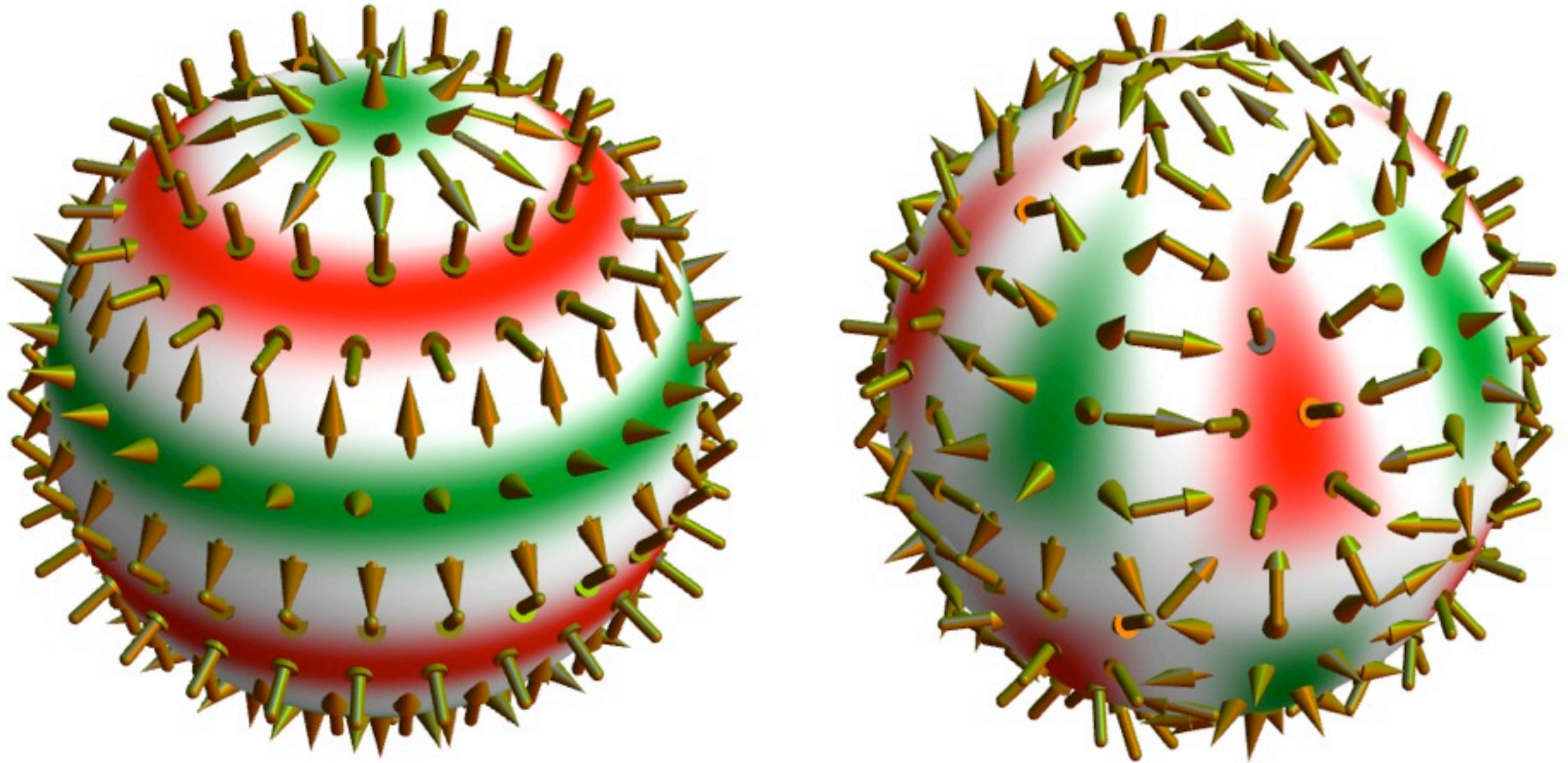
Pressure: mainly  
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Divergence in  
615 exchange  
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# Triakontadipole moments

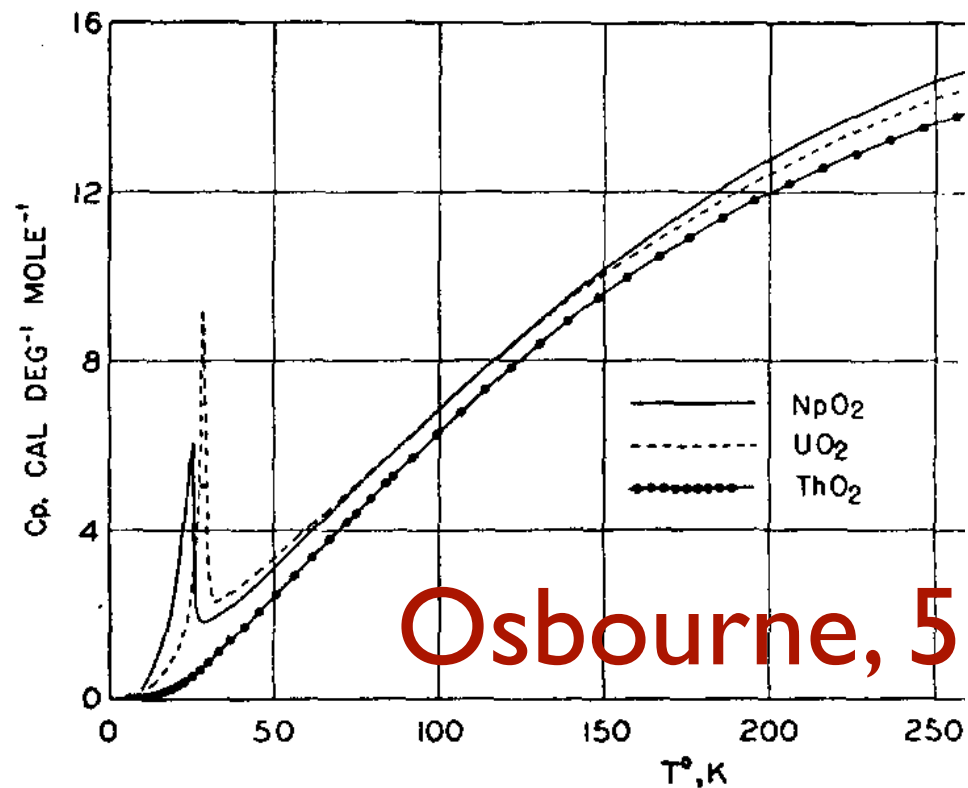


## The hidden order

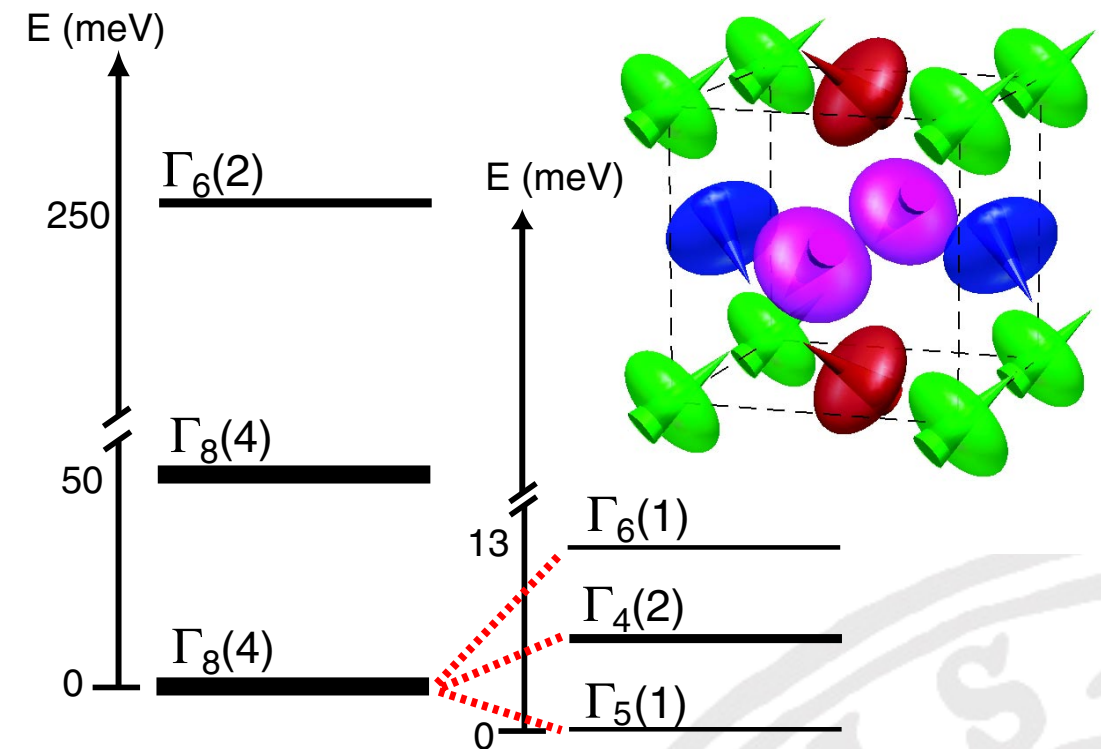
Cricchio *et al.* PRL **103**, 107202 (09)



# Hidden order: $\text{NpO}_2$



Osbourne, 53



Magnani et al, 08

- ✱ 2<sup>nd</sup> order transition at  $T_o=26$  K
- ✱ but no moments
- ✱ AF-I  $3\mathbf{k}$  type ordering
- ✱ Suggestions: rank 2, 3 and 5 ...
- ✱ Triakontadipoles from localized CF picture
- ✱ consistent with INS

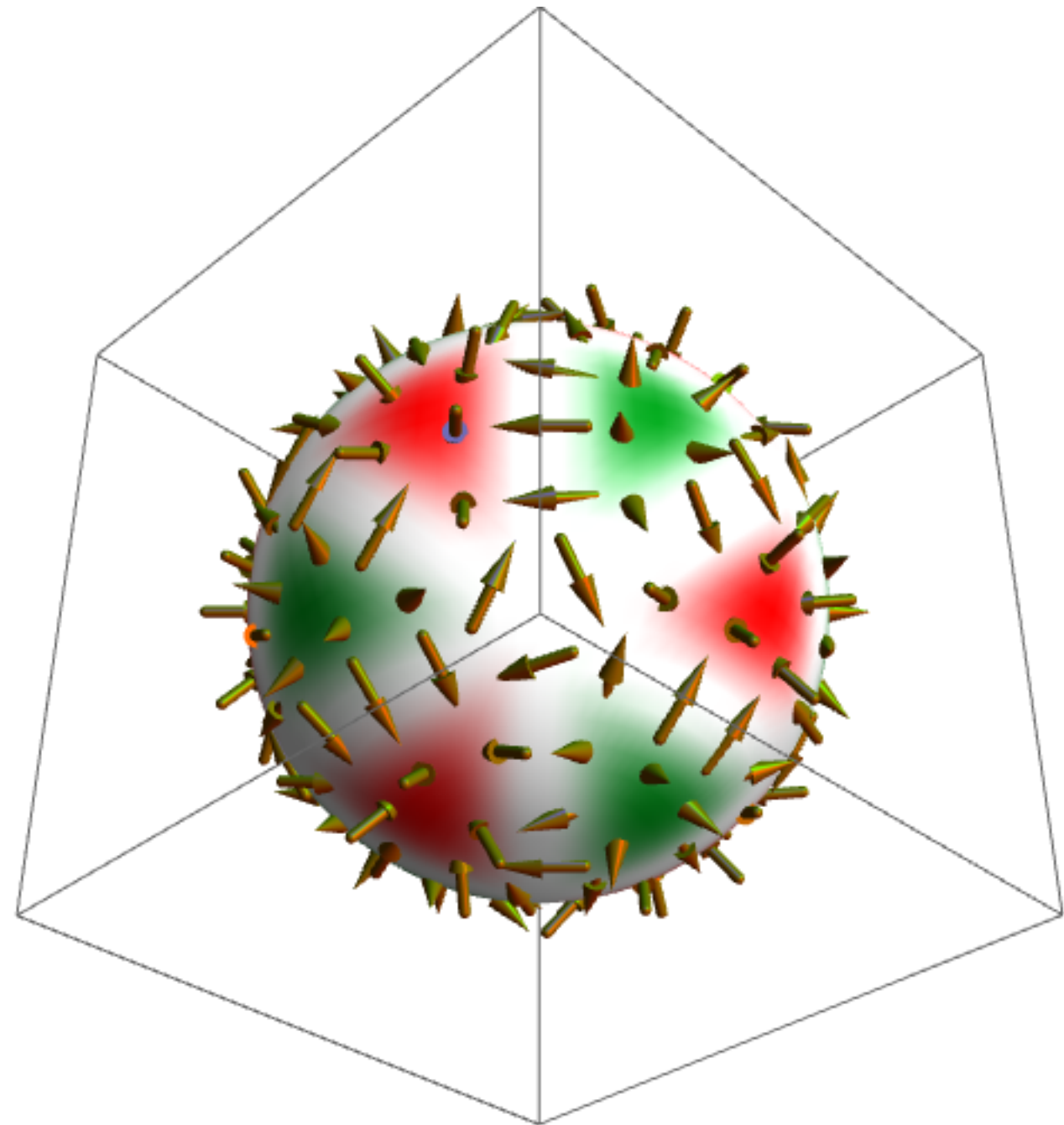
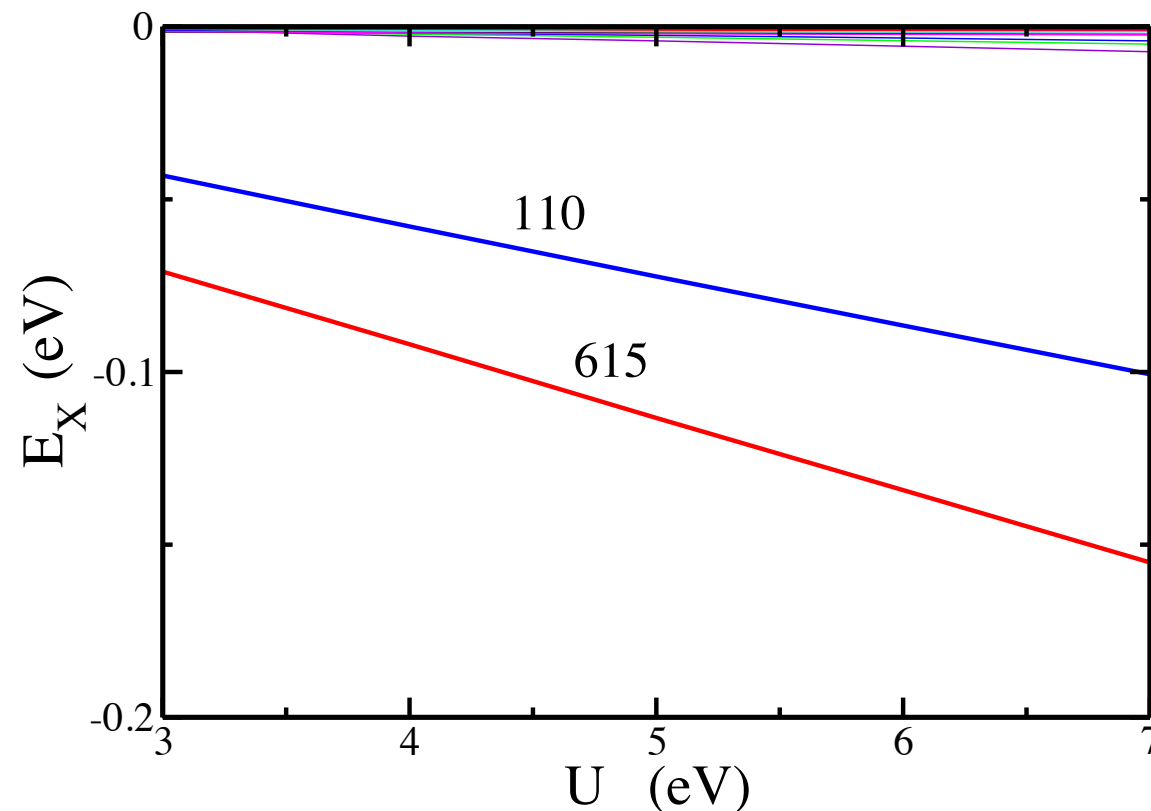
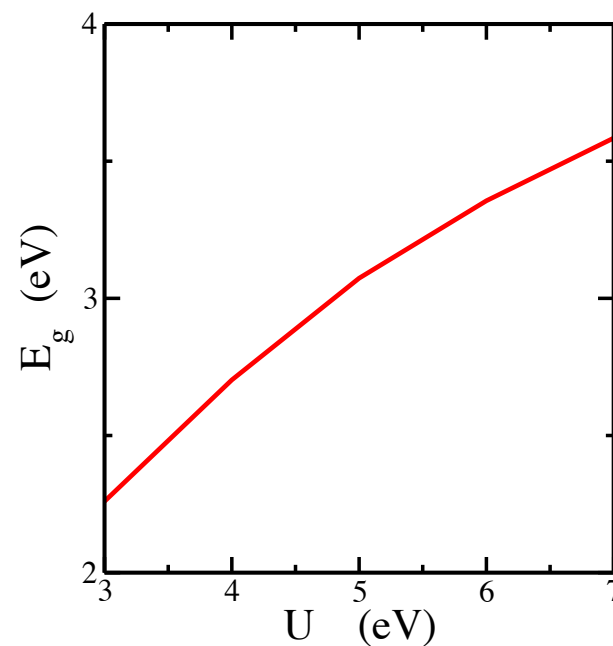




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# Hidden order: $\text{NpO}_2$

LDA+ $U$ :  
3k lowest  
insulator  
no moment

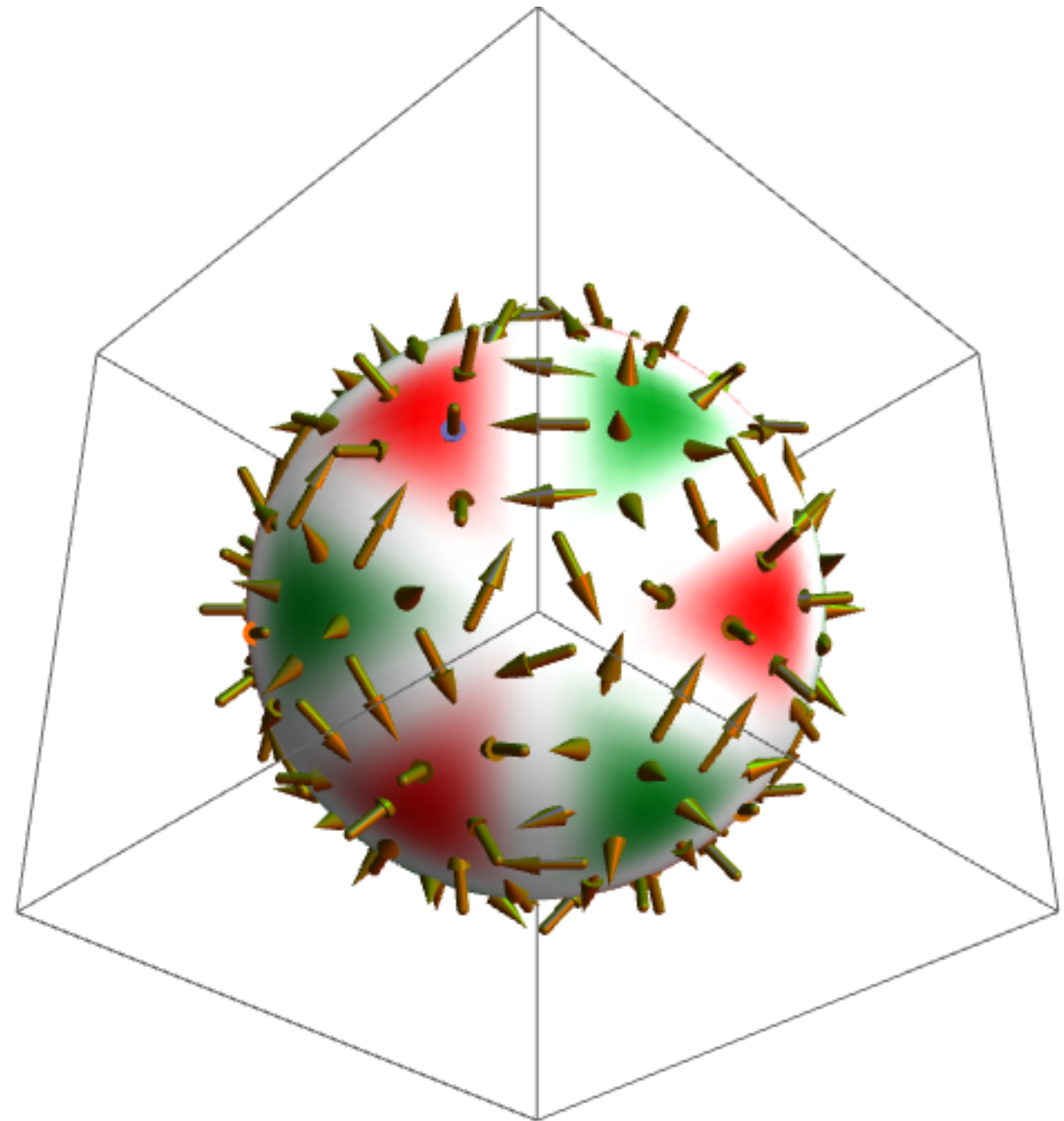
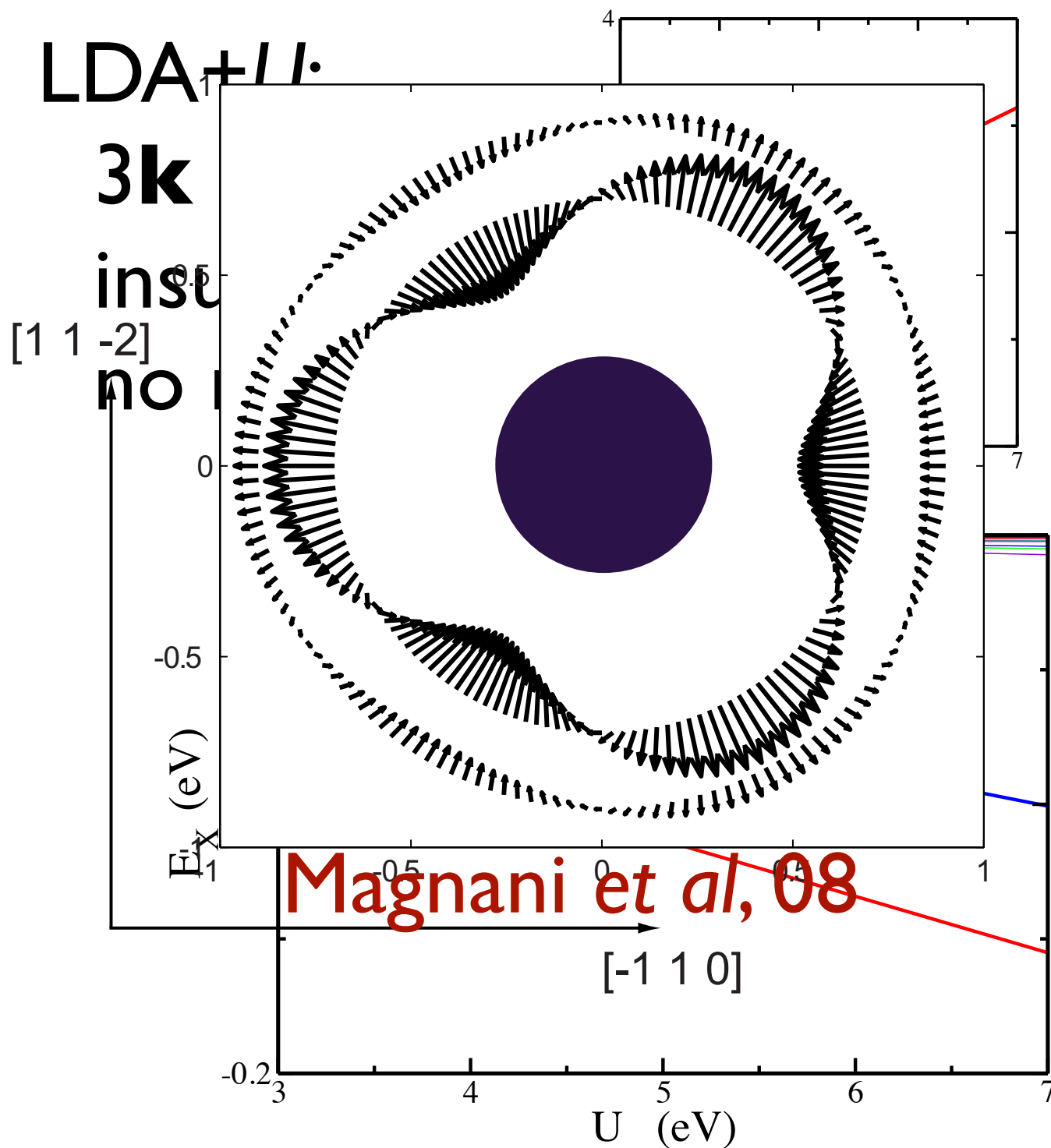


local magnetization at  
one of four sites



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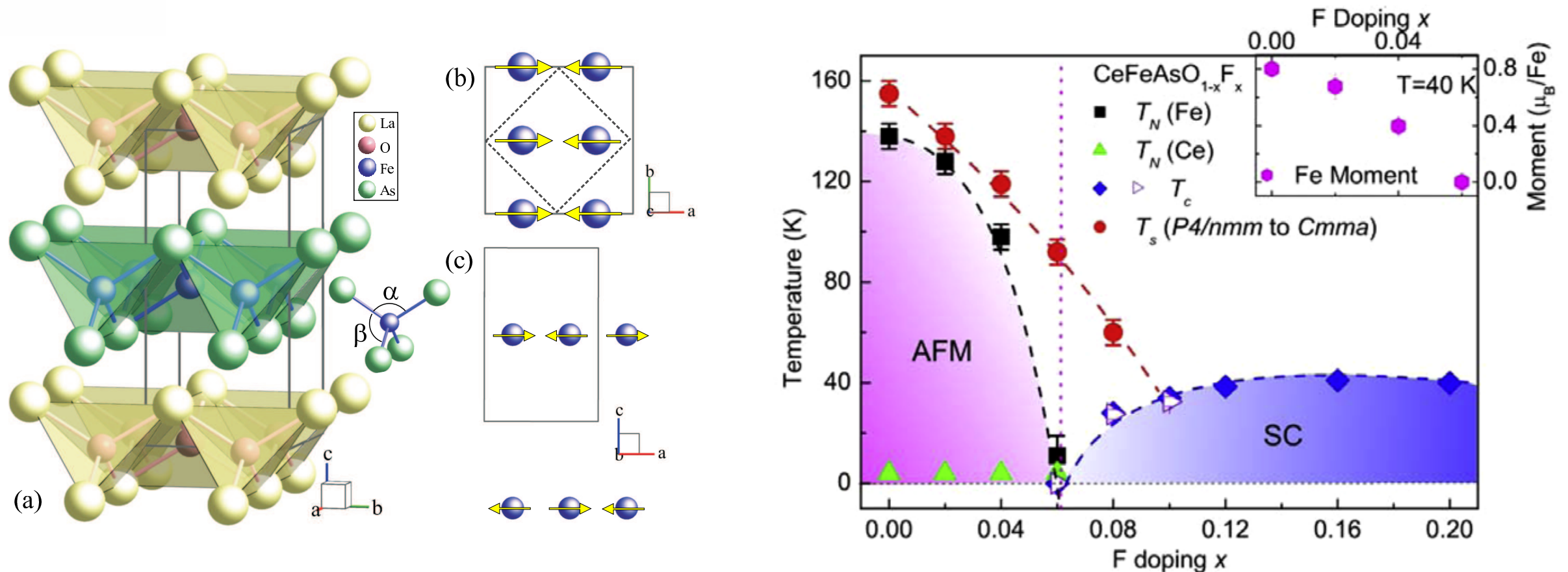
# Hidden order: $\text{NpO}_2$



local magnetization at  
one of four sites



# Fe-pnictides



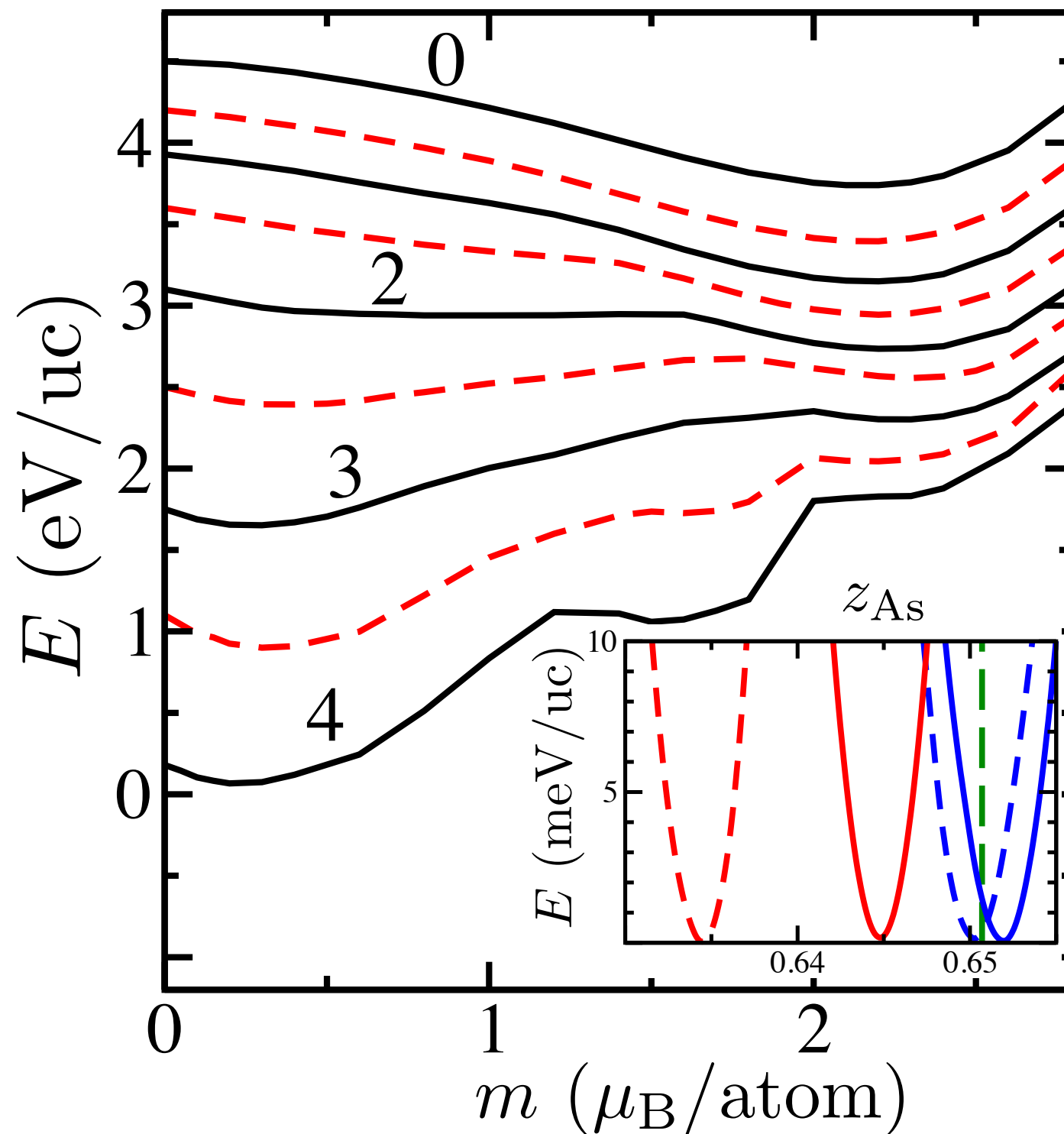
- ✱ FeAs layers
- ✱ SC under doping – up to 60 K ...
- ✱ AF in parent compounds
- ✱ Correlated electrons?
  - ✱ Not understood: Small moments & FeAs bonds



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# Small moments

DFT+ $U$



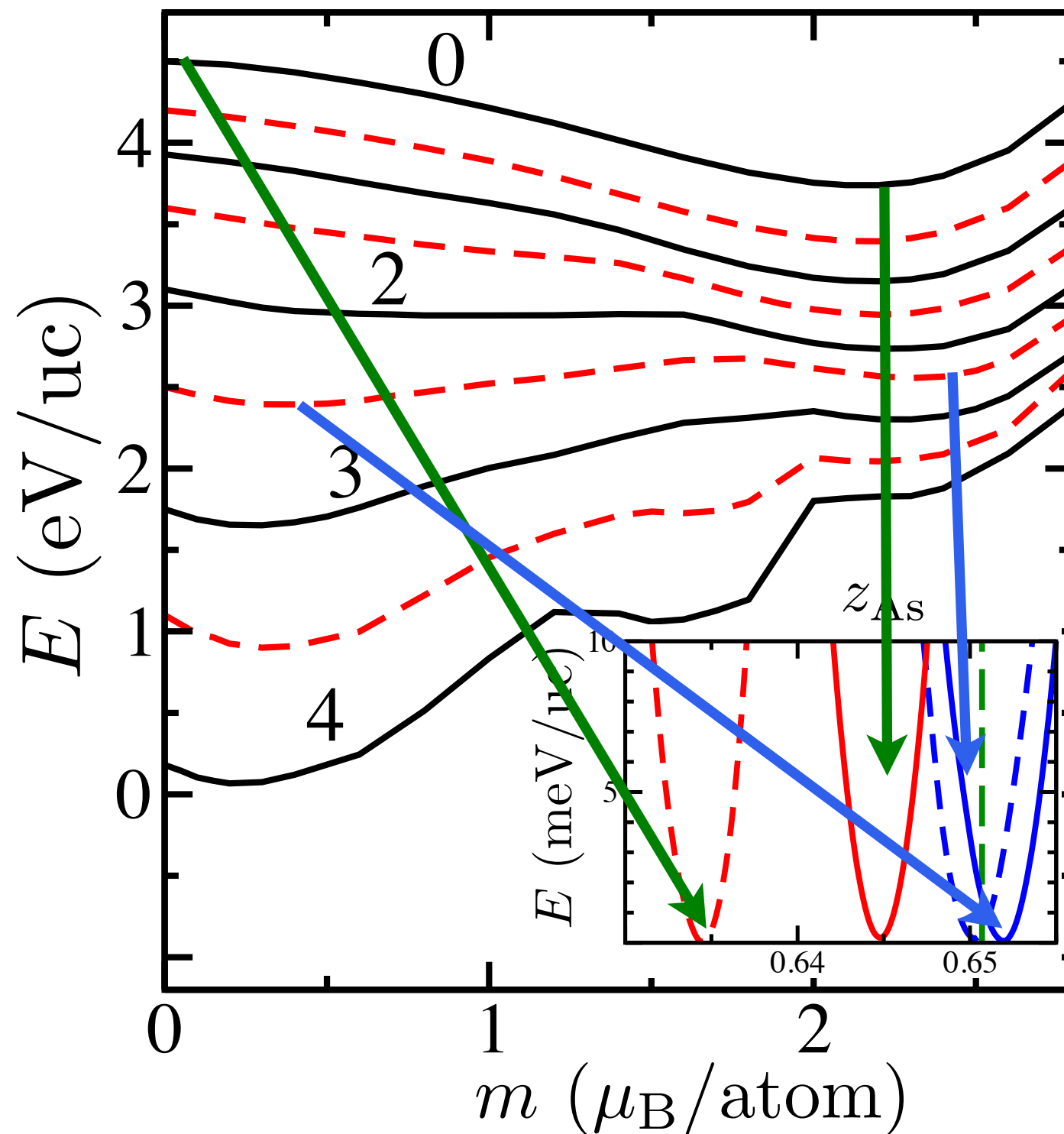




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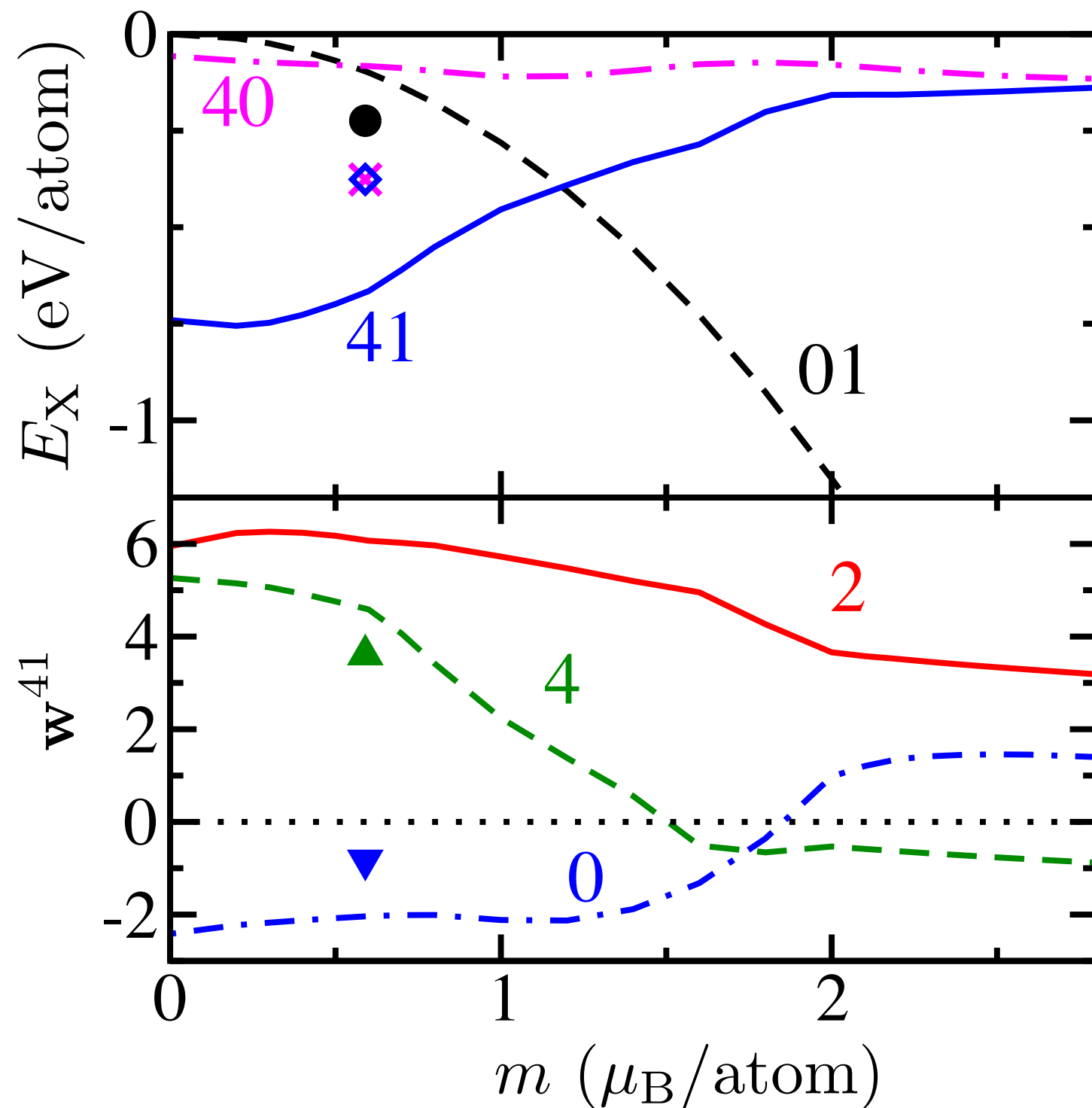
# Small moments

DFT+ $U$





# Tensor moments





# Contribution to DM

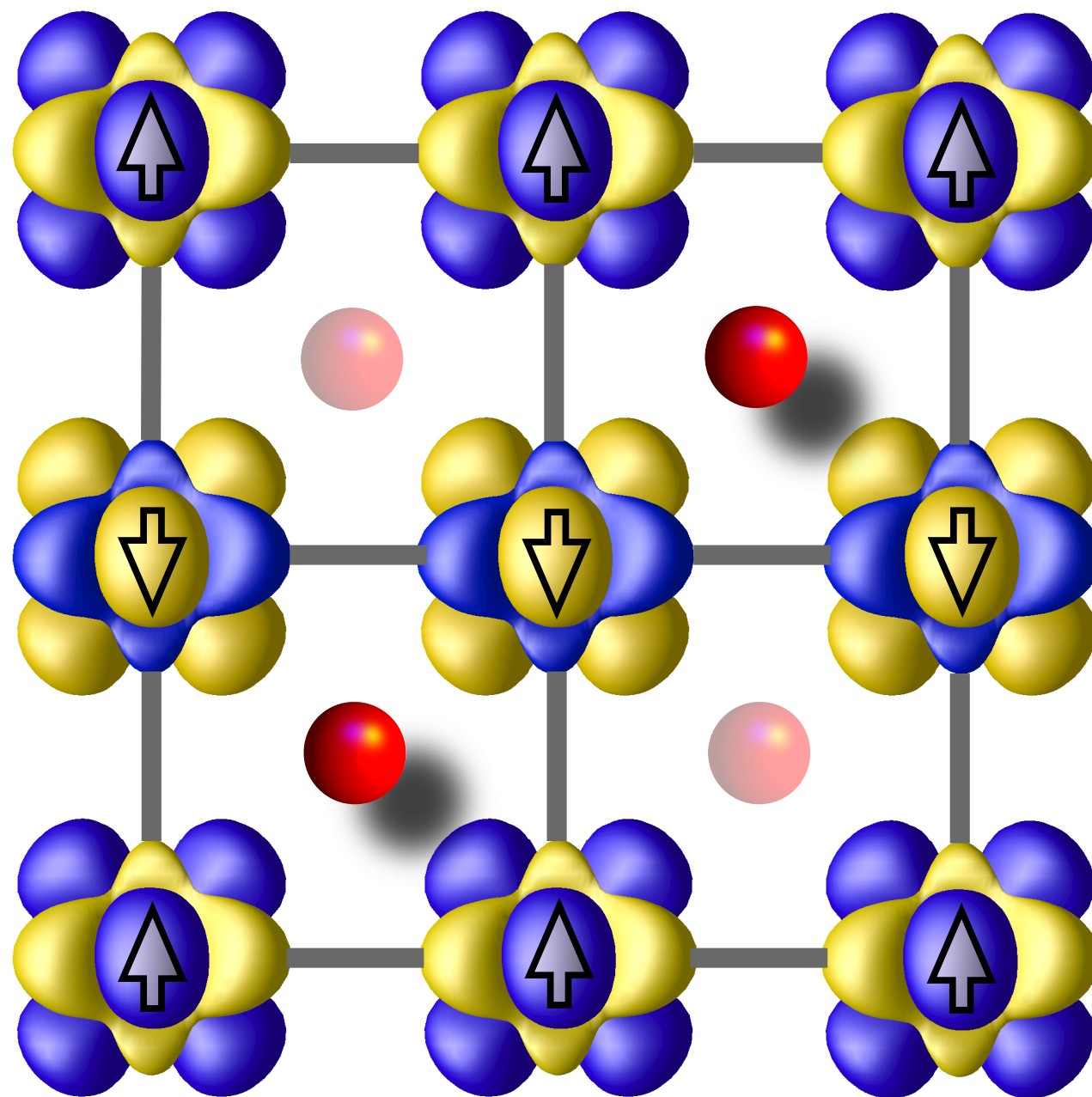
$$\Gamma_{20}^{41} = \begin{pmatrix} 2\sqrt{5} & 0 & 0 & 0 & 0 \\ 0 & -2\sqrt{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{15} \\ 0 & 0 & 0 & \sqrt{15} & 0 \end{pmatrix} \otimes \sigma_z$$

$xz \quad zy \quad xy \quad x^2 - y^2 \quad z^2$



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# Magnetization density



Cricchio *et al.* PRB **81**, 140403(R) (10)



# Summary

- Brief description of DFT+ $U$
- Reformulated (scr) exchange – tensor moments
- Unexpected exchange channels dominate
- Hidden order in  $\text{URu}_2\text{Si}_2$  &  $\text{NpO}_2$  – triakontadipole
  - Katt's rules to substitute Hund's in actinides
- Multipoles play a role in undoped FeAs
  - Small moments
  - FeAs bonding distance

# Epilogue

- What about Kondo and Mott and these guys ...?
- Dynamic effects are important
- Our static MF approach overestimates the effect
- BUT
  - The energies involved are large
  - Agreement with experiments is compelling
    - Reduced spin moments & Hidden order etc





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The Swedish Research Council

# Collaborators

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